KNOT THEORY

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ST. PAUL'S COLLEGE, KALAMASSERY
(AFFILIATED TO M G UNIVERSITY, KOTTAYAM)

CERTIFICATE

This is to certify that the project report titled "KNOT THEORY" submitted by SHARVIN BENEDICT (Reg no: 170021032430), CHRISTI PAPPACHAN (Reg no: 170021032404), ANEESH ANTONY (Reg no: 170021032396)towards partial fulfilment of the requirements for the award of Degree of Bachelor of Science in Mathematics is a bonafide work carried out by them during the academic year 2017-2020.

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Place: Kalamassery
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Examiner :

DECLARATION

We , submitted by SHARVIN BENEDICT (Reg no: 170021032430),CHRISTI PAPPACHAN(Reg no: 170021032404),ANEESH ANTONY (Reg no: 170021032396) hereby declare that this project entitled "KNOT THEORY" is an original work done by us under the supervision and guidance of Ms. Maya K, faculty, Department of Mathematics in St. Paul's college Kalamassery in partial fulfilment for the award of The Degree of Bachelor of Science in Mathematics under Mahatma Gandhi University. We further declare that this project is not partly or wholly submitted for any other purpose and the data included in the project is collected from various sources and are true to the best of our knowledge.

Place: Kalamassery

SHARVIN BENEDICT

Date:

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ACKNOWLEDGEMENT

For any accomplishment or achievement, the prime requisite is the blessing of the Almighty and it's the same that made this world possible. We bow to the lord with a grateful heart and prayerful mind.

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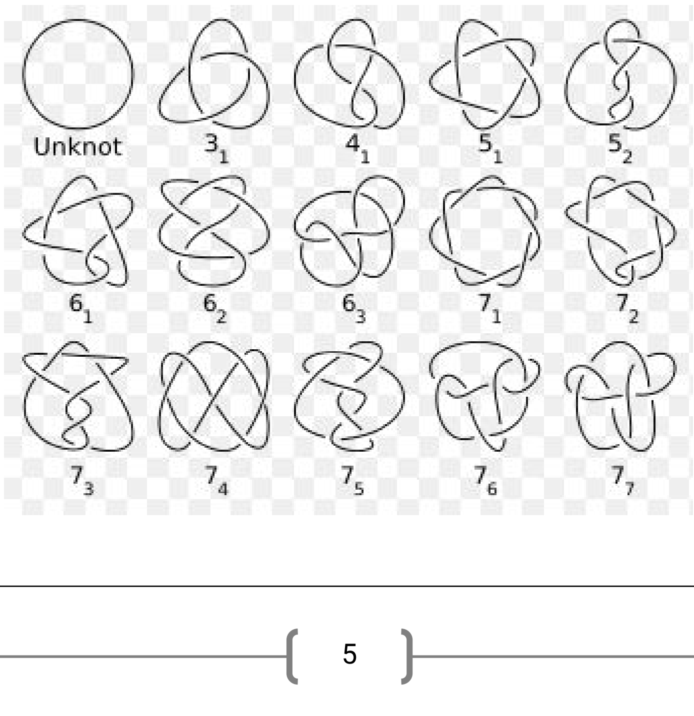
CONTENT

INTRODUCTION

In topology, knot theory is the study of mathematical knots while inspired knots which appear in daily life. Such as those in shoelaces and rope, a mathematical knot differs in the ends are joined together so that it cannot be done. Take a piece of string, tie a knot in it. Now glue the ends of the string together to form a knotted loop. The result is a string that has no loose ends and that is truly knotted. Unless we use scissors there is no way that we can untangle the string.

To make a study of knottedness, the knotted part of the string must be trapped. One way to do this is to imagine an infinitely long string which is a straight line outside the region containing the knot. A simple way to join the ends to form a loop in mathematical language, a knot is an embedding of a circle in three dimensional space. Mathematical knots are modelled on the physical variety and we allow to knot to be deformed as if it were made of a thin, flexible, elastic thread.



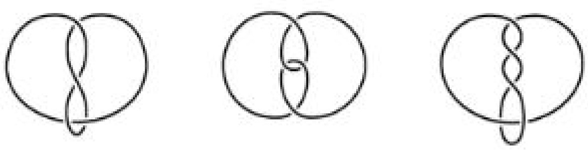
CHAPTER 1

ELEMENTARY CONCEPTS

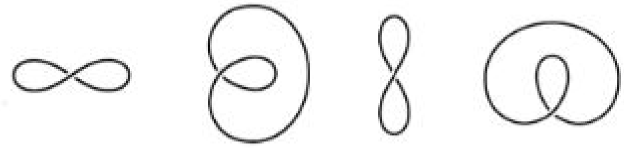
WHAT ARE KNOTS?

A knot is just such a knotted loop of string except that we think of the string as having no thickness, its cross section being a single point. The knot is then a closed curve in space that does not interact itself anywhere. Deformation of curve will be considered to be the same knot.

The simplest knot of all is just the unknotted circle, which we call the Unknot or the Trivial knot. The next simplest knot is called a Trefoil knot. all of the knots are known to be distinct .If we made any one of them out of string, we would not be able to deform it to look like any of the others .there are many different pictures of the same knots. We call such a picture of knot a projection of the knot.



The places where the knot crosses itself in the picture are called the crossings of the projections, figure-eight knot is a four-crossing knot. If a knot is to be non -trivial, then it had better have more than one crossing in a projection.



Because each of these is clearly a trivial Knot as we can then untwist the single crossing interestingly enough, there are no two crossing non trivial Knot.

Knot theory is a subfield of an area of mathematics known as topology.

Certain types of knots are particularly interesting. One such type is an alternating knot, which is a knot with a projection that has crossing that alternate between over and under as one travels around the knot in a fixed direction.

E.g. - Trefoil Knot

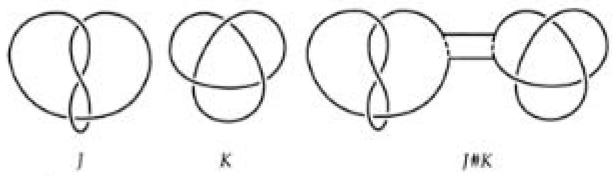
COMPOSITION OF KNOTS

Given two projections of knots, we can define a new knot obtained by removing a small arc from each knot projection and then connecting the four end points by two new arcs. We call the resulting knot the composition of the two knots. If we denote the two knots by the symbols J and K, then their composition is denoted by J#K.

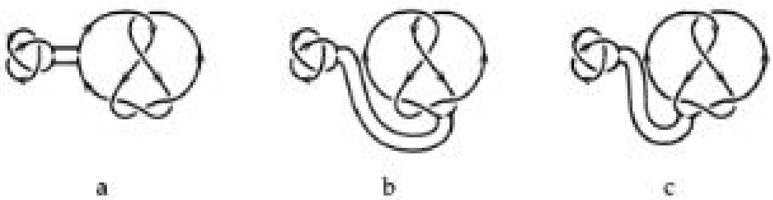
We choose the two arcs that we remove to be on the outside of each projection and to avoid any crossings. We choose the 2 new arcs so they do not cross either the original knot projections or each other.

We call a knot a composite knot if it can be expressed as the composition of 2 knots, neither of which is trivial knot. This is in analogy to the positive integers, neither of which is equal to1.The knots that make up the composite knot are called factor knots. If a knot is not the composition of any two nontrivial knots, we call it a prime knot.

E.g. -trefoil knot, figure-eight knot.



One way that composition of knots does differ from multiplication of integers is that there is more than one way to take the composition of two knots .it is often possible to composition of two knots .it is often possible to construct two different composite knots from the same pair of knots J and K . We first need to put an orientation on our knots. An orientation is defined by choosing a direction to travel around the knot. Thus direction is denoted by placing coherently directed arrows along the projection of the knot in the direction of our choice. We then say that the knot is oriented.



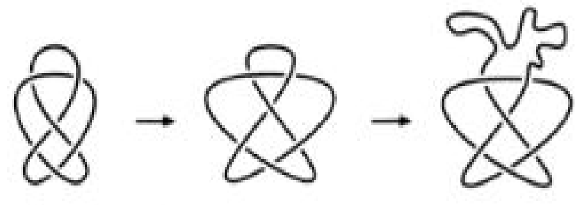
When we then form the composition of two oriented knots J and K, there are two possibilities. Either the orientation on J matches the orientation on K in J#K, resulting in an orientation for J#K, or the orientation J and K do not match up in J#K .All of the compositions of the two knots where the orientations do match up will yield the same composite knot, and vice versa.

A knot is invertible if it can be deformed back to itself so that an orientation on it is send to the opposite orientation.

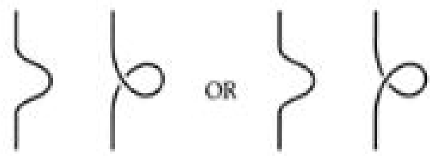
REIDEMEISTER MOVES

Suppose that we have two projections of the same knot. If we made a knot out of string that modelled the first of the two projection that we should be able to rearrange the string to resemble the second projection. Knot theorists call the rearranging of the string, that is, the movement of the string through tree-dimensional space without letting it pass through itself, an ambient isotopy.

A deformation of a knot projection is called a planar isotopy if it deforms the projection plane as if it were made of rubber with the projection drawn here.



A reidemeister move is one of the three ways to change a projection of the knot that will change a projection of the knot that will change a projection of the knot that will change the relation between the crossing .The first reidemeister move allows us to put in or take out a twist in the knot, as in the given figure.

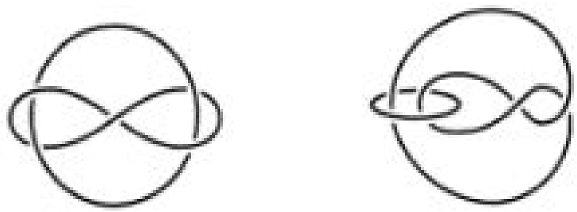


The second reidemeister move allows us to either add two crossings or remove two crossings. The third reidemeister move allows us to slide a strand of the knot from one side of a crossing to the other side of the crossing

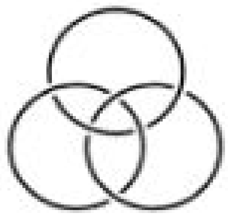
The figure-eight knot is known to be equivalent to its mirror image, that is, the knot obtained by changing every crossing. Incidentally, a knot that is equivalent to its mirror image is called amphicheiral by mathematicians and achiral by chemists.

LINKS

A link is a set of knotted loops all tangled up together. Two links are considered to be the same if we can deform the one link to the other link without ever having any one of the loops intersect itself or any of the other loops in the process. Here are 2 projections of one of simplest links, known as the whitehead link.



Since it is made up of two loops knotted with each other, we say that it is a link of two components. Here is another well-known link with three components, called the Borromean rings. This links is named after the Borromeas, an Italian family from the renaissance that used this pattern of interlocking rings on their family crest.



We call the first of these unlink (or trivial link) of two components and the second the Hopf link. One difference between these two links is that the unlink is splittable since its two components don't link each other and can be separated by a plane. But in the hopf link, the two components do link each other once. We would link a method for measuring numerically how linked up two components are. We will define what's known as linking number

We say that the linking number is an invariant of the oriented link that is once the orientations are chosen on the two components of the link, the linking number is unchanged by ambient isotopy. It remains invariant when the projection of the link is altered. This is one of many invariants we will look. Another invariant of links is simply the number of components in the link. It is unchanged by ambient isotopes of the link.

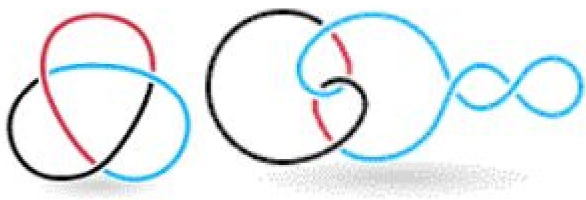
A link is called Brunnian if the link itself as non-trivial, but the removal of any of the components leaves us with a set of trivial unlinked circles. These links are named after Hermann Brunn, who drew pictures of such links in 1892.

TRICOLORABILITY

We have discussed a lot about telling knots and links apart, but actually we have not yet shown the most basic fact of knot theory. We have not yet proved that there is any other knot besides the unknot. So we will prove that there is at least one other knot besides the unknot. We will prove that the trefoil knot is not equivalent to the unknot. In order to do that, we need to introduce the idea of tricolorability.

We will say that a strand in a projection of a link is a piece of the link that goes from one undercrossing to another with only overcrossing in between. We will say that a projection of a knot or link is tricolorable if each of the strands in projection can be colored one of three different colors, so that at each crossing either three different colors come together or all the same color comes together. In order that a projection be tricolourable, we further require that at least two of the colors are used.

Since Reidemeister moves leave the colorability unaffected, whether or not a projection is tricolorable depends only on the knot given by the projection. Either every projection of a knot is tricolorable or no projection of that knot is tricolorable. For instance, every projection of the trefoil knot is tricolorable. Since the usual projection of the unknot is not tricolorable, it must be the case that the trefoil knot and the unknot are distinct.



CHAPTER 2

TABULATING KNOTS

- THE DOWKER NOTATION FOR KNOTS

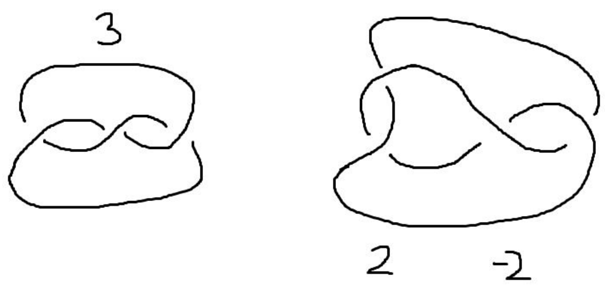
The Dowker notation is an extremely simple way to describe a projection of a knot. First, start with an alternating knot. Suppose we have a projection of alternating knot that we want to describe. Choose an orientation on the knot, given by placing coherently directed arrows along the knot. Pick any crossing and label it 1. Leaving that crossing along the under strand in the direction of the orientation, label the next crossing that you come to with a2. Continue through that crossing on the same strand of the knot, and label the next crossing with a3. Continue to label the crossing with the integers in sequence until you have gone all the way around the knot once when you are done, each crossing will have two labels on it as the knot passes through each crossing twice. Each crossing has one even number and one odd number labelling it.

- CONWAY'S NOTATION

We introduce a notation for knots due to John H Conway. This was the notation he used in order to tabulate the prime knots through 11 crossing and prime links through 10 crossing in 1969. The Conway notation has been utilized in order to prove numbers results and recently has been applied to knotting in DNA. It is particularly suited to calculations involving what are called tangles.

A tangle in a knot or link projection is a region in the projection plane surrounded by a circle such that the knot or link crosses the circle exactly four times. We will say two tangles are equivalent if we can get from one to the other by a Reidemeister move sequence while the four endpoints of the strings in the tangle remain fixed and while the strings of the tangle never journey outside the circle defining the tangle. If the rational tangle is represented by an even number of integers, we can think of constructing it by simple starting with two vertical strings and then twisting the two bottom end points around each other some number of times , while holding the top two endpoints fixed .similarly if the rational tangle is represented by an odd number of integers , we can construct it by starting with two horizontal strings and alternately twisting the two right-hand endpoints appropriately, followed by twisting the two bottom endpoints appropriately.

Amazingly enough, there is an extremely simple way to tell if two rational tangles are equivalent. Suppose that two tangles are given by the sequences of integers -2 3 2 and 3 -2 3. We compute the so called continued fractions corresponding to these integers .If we close off the ends of a rational tangle, we call the resulting link a rational link. So for instance, the figure -eight knot is a rational knot, with rational tangle 22. We can use our notation for rational tangles to denote the corresponding rational knot. We call this notation Conway's notation.

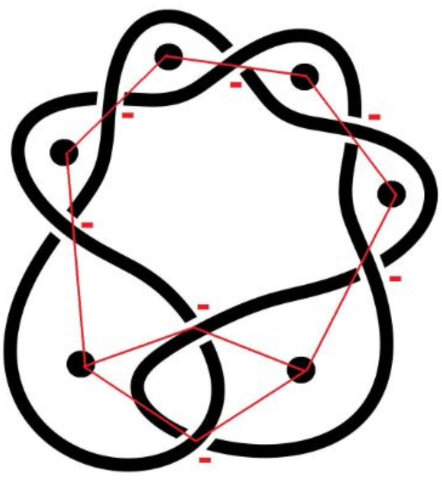


- <u>KNOTS AND PLANAR GRAPHS</u>

We introduce a notation for knot projections that has been useful in the past for knot tabulation. It provides a bridge between knot theory and graph theory, with the potential for commerce in both directions. Here we are interested in planar graphs, that is, graphs that lie in the plane. From a projection of a knot or link, we create a corresponding planar graph in the following way. First shade every other region of the link projection so that the infinite outermost region is not shaded.

Put a vertex at the centre of each shaded region and then connect with an edge any two vertices that are in regions that share a crossing. This is the graph corresponding to our projection. It doesn't depend in any way on whether a crossing is an over crossing or an undercrossing. So we define crossings to be positive or negative. Now we label each edge in the planar graph with a + or a – depending on whether the edge passes through + or – crossing. We call the result a signed planar graph. We now have a way to turn any link projection into a signed planar graph.

Certainly, we can turn any signed planar graph into a knot projection. Starting with the signed planar graph put an x across each edge. Connect the edges inside each region of the graph. Shade those areas that contain a vertex. Then, at each x's put in a crossing corresponding to whether the edge is a + or a – edge. The result is a link.



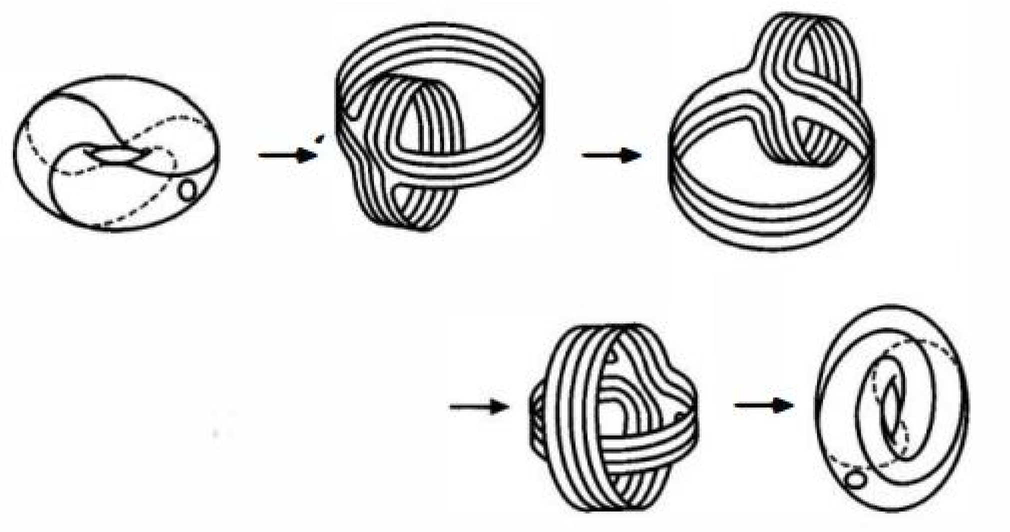
CHAPTER 3

TYPES OF KNOTS

- TORUS KNOT

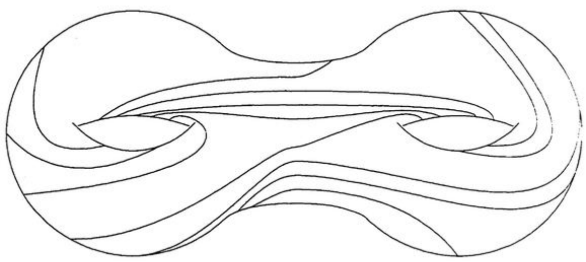
We call a curve that runs once the short way around the torus a meridian curve. A curve that runs once around the torus the long way is called a longitude curve. The trefoil knot wraps three times meridionally around the torus and twice longitudinally. Every torus knot is a (p, q)-torus knot for some pair of integers. In fact, the two integers will always be relatively prime.

If we want to draw a (p, q)-torus knot, we just place p points around the inside and outside equators of the torus, attach the inside and outside points directly across the bottom of the torus, and then attach each outside point to the inside point that is clockwise q points ahead, using a strand that goes over the top of the torus. In fact, every (p, q) - torus knot is also a (q, p)-torus knot. Say for instance that we have the trefoil knot, which we have seen is a (3, 2)-torus knot. A (p, q)-torus knot has a projection with p (q -1) crossing and a projection with q (p -1) crossings. Therefore, the crossing number for a (p, q) - torus knot is at most the smaller of p(q-1) and q(p-1) . It has recently been proved by KunioMurasugi of the university of Toronto that in fact the smaller of p(q -1) and q(p-1) is exactly the crossing number of a (p, q)-torus knot .



A solid torus is a doughnut where we include both the interior of the doughnut as well as the surface. The core curve of a solid torus is the trivial knot that runs once around the centre of the doughnut. A meridionally disk of the solid torus is a disk in the solid torus that has a meridian curve as its boundary.

We can generalize the notation of a torus knot. By definition, a torus knot is a non-trivial knot that can be placed on the surface of a standardly embedded torus without crossing over or under itself on the surface. By standardly embedded, we mean that the torus is unknotted in space. But certainly, there will be knots that cannot be placed on a standardly embedded torus but that can be placed on a standardly embedded genus two surface.

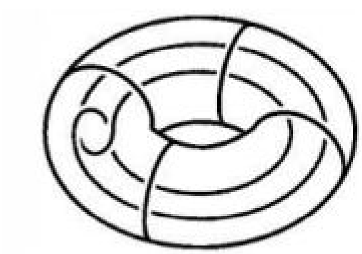


For lack of a better name, let’s call these 2- embedded knots since they can be standardly embedded genus two surface. For instance, the figure

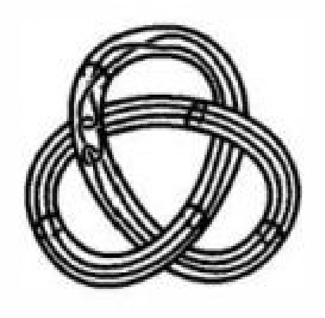
-eight knot is a 2-embeddable knot. More generally, we will say that a knot K is an n-embeddable knot if K can be placed on a genus n-standardly embedded surface without crossing, but K cannot be placed on any standardly embedded surface of lower genus without crossings.

- <u>SATELLITE KNOTS</u>

A second set of knot that has become very important in recent years is the set of satellite knots. Let K_1 be a knot inside an unknotted solid torus. We note that solid torus in the shape of a second knot K_2. This will take the knot K_1 that lies inside the original solid torus to a new knot inside the knotted solid torus. We call this new knot K_3, a satellite knot. The knot K_2 is called the companion knot of the satellite knot. We always assume that the companion knot is a non-trivial knot, since otherwise the resulting satellite knot would just be K_1 back again. We also always assume that the knot K_1 hits every meridional disk of the solid torus, and it cannot be isotoped to miss any of them. We think of the satellite knot as a knot that stays within a solid torus that has a companion knot as its core curve, just as a satellite stays within orbit around a planet.



There is knotted torus in space that misses the satellite knot, lying in the compliment of the knot. In fact, this knotted torus is always an incompressible, but proving this will take a substantial amount of work. If on the other hand, we take the original knot K_1 to be an unknot, but sitting inside the solid torus twisted up, then the resulting satellite knot is called whitehead double of the companion knot. The name refers to the fact that the knot K_1 here resembles the white head link.



If the original knot K_1 is again unknotted , but sitting inside the solid torus then the resulting satellite knot is called two-strand cable of the companion knot .It's as if we had a cable that ran twice around the companion knot .Again ,the two-strand cable will not be unique, as we can add twists to it.

The operation of forming a satellite knot can be thought of as a generalization of the idea of composition. If K_1 only has one strand that reaches longitudinally around the solid torus, then the satellite knot formed by knotting the solid torus like K_2 is in fact the composite knot $K_1 \# K_2$.

If the knot K_1 that we start with is a torus knot, then we call the resulting satellite knot with companion K_2 a cable knot on K_2. We can think of it as taking a cable that wraps around the knot K_2 a total of p times meridionally and q times longitudinally. In one field of mathematics called algebraic geometry, the most prevalent types of knots are cable knots. Sometimes the cable knots are cables on torus knots.

- <u>HYPERBOLIC KNOTS</u>

A hyperbolic knot is a knot that has a complement that can be given a metric of constant curvature -1. Usually, we measure the distance between two points $P_0(x_0, y_0, z_0)$ and $P_1(x_1,y_1,z_1)$ in three space using the formula

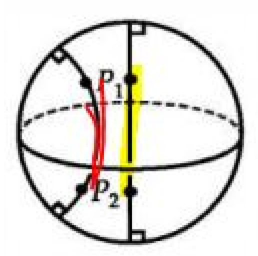
$d(P_0,P_1) =\sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$

This method for measuring distance is called the Euclidean metric.

We are interested in the 3-dimentional space, so we can't draw the pictures like we could of the sphere, plane and saddle. But the Euclidean metric for 3-space that we gave before is an example of a metric with curvature zero. It is so called flat metric, having no curvature, just like the plane is flat. The metric that we want to put on the compliment of the knot is not flat, but rather has curvature -1.The geometry that results is called the hyperbolic geometry and the metric is called hyperbolic metric. The hyperbolic 3-space, H^3, is the simplest example of the 3-dimensional space that has a hyperbolic metric. Any arc of a circle or diameter in H^3 that is perpendicular to the unit sphere is called a geodesic in H^3.

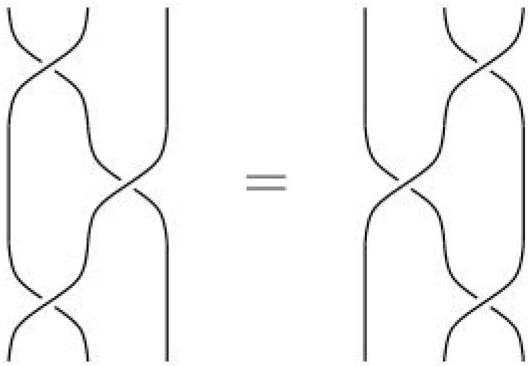
We can use the hyperbolic method for measuring distance within the individual tetrahedral in order to obtain a hyperbolic method for measuring distance in the entire knot compliment. We then say that the knot is hyperbolic knot.

Every hyperbolic knot has a hyperbolic volume. This is a positive real number that can be computed out to as many decimal places are needed. It is simply the sum of volumes of individual hyperbolic tetrahedra that make up the knot compliment of the knot, as measured by our hyperbolic metric. Although it appears that the volume of 3-space minus the knot would be infinite, it is in fact finite when we measure it using this hyperbolic method of measuring volume. The hyperbolic volume is an invariant for the hyperbolic knots, as it depends only on knot itself and not on any particular projection of knot.



- <u>BRAIDS</u>

Braids are not a particular type of knot. However every knot can be describe by a braid. A braid is a set of n strings, all of which are attached to a horizontal bar at the top and at the bottom.



Each strings always head downwards as we move along any one of the strings from the top bar to the bottom bar. Another way to say the same thing is that each string intersects any horizontal plane between the two bars exactly once. We can always pull the bottom bar around and glue it to the top bar, so that the resulting strings form a knot or link called the closure of the braid. Therefore every braid corresponds to a particular knot or link .we can think of there being an axis coming right out of the page, around which the closure of the braid is wrapped. We then have a closed braid representation of the knot if there is a choice of orientation on the knot so that, as we traverse the knot in that direction, we always travel clockwise around the axis without any backtracking.

We can see two projections of the trefoil with axes, one of which is not a closed braid around its axis, and the other of which is a closed braid around its axis

Knots and links can be represented as closed braids every knot or link is a closed braid .this was first proved by J.W. Alexander in 1923.

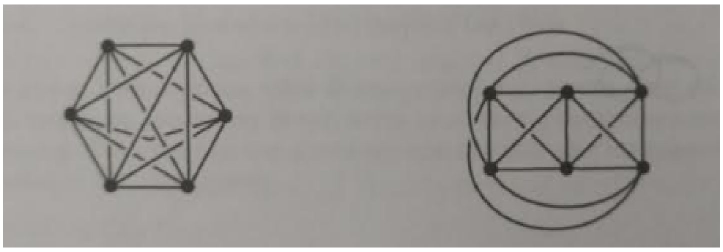
Markov's theorem says that two braids Markov equivalent if and only if they are related through a sequence of the three operations that we have already seen, which are the operations that obviously give us back the same open braid, and two additional operations. The first operation is called the conjugation. The next operation is called stabilization.

CHAPTER 4

KNOTS, LINKS AND GRAPHS

- LINKS IN GRAPHS

The graph K6, called the complete graph on six vertices, is the graph where every one of the six vertices is connected to every other one by exactly on edge.



Although these two graphs are isomorphic, they are not isotopic, since there is no way to deform one of them through spaces to look like the other, without allowing edges to pass through themselves or each other. We call a partition way to place K_6 in space, an embedding of K_6.

Let's call a triangle in an embedding of K_6 any set of three consecutive edges that form a triangle in the graph. If we choose any three vertices, we can form a triangle from the edges connecting them. We can also form a second triangle from the remaining three vertices.

Every embedding of K_6 contains at least one pair of linked triangles. No matter how we place K_6 in space, there will be always a link contained within it. Even if we change the embedding by letting one edge pass through another specifically in order to destroy a link in the original embedding, we can’t help but either create a new link in the process or at least leave another link in the embedding.

We need a way to distinguish embeddings, a so-called invariant for the embedding. Suppose we have a particular embedding of K_6 .Each pair of disjoint triangles in the embedding has a linking number once we orient the two triangles. But, changing once an orientation on one of the triangles only changes the sign of the linking number, not the absolute value of the linking number .Since we don't want to bother with orientations, we just look at the absolute values of the linking number for each pair of disjoint triangles in the embedding.

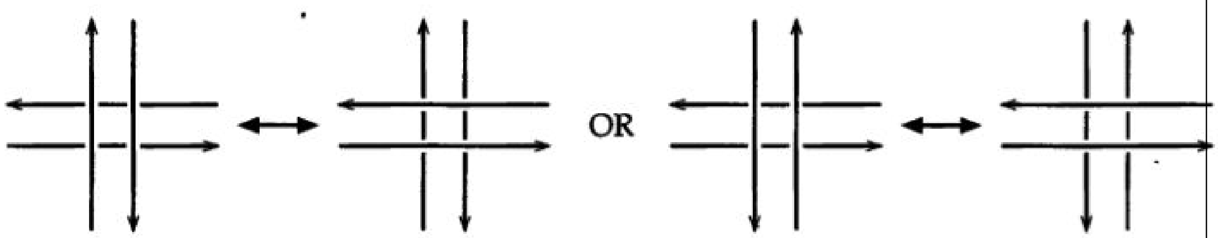
Any graph that containing K_6 as the sub graph will also contain a link in any embedding of it into three-space. We say that a graph is intrinsically linked. If it has the property that any embedding of it in three-space contains a nontrivial link.

We define an expansion of a graph G to be a new graph obtained from g by splitting a vertex of G. By this we mean replacing a particular vertex v of G by two vertices u and w connected by a new edge, and replacing each of the old edges that ended at v by a new edge that begins where the old edge began and ends at either u or w. There are lots of choices for expansions even if we have already chosen the vertex to expand. If G is intrinsically linked, so is any expansion of G.

- <u>KNOTS IN GRAPHS.</u>

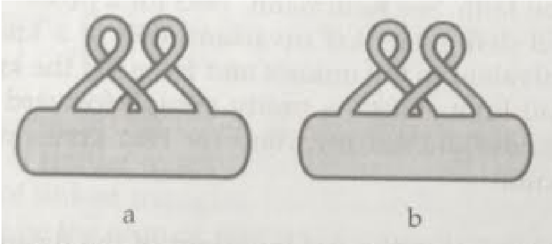
A Hamiltonian cycle in a graph is a sequence of edges in the graph such that any two consecutive edges share a vertex, the last edge and the first edge share a vertex, and every vertex is hit by a pair of consecutive edges exactly once .Together the edges in the Hamiltonian cycle make up a loop in the graph that take hits every vertex exactly once. Such a loop may be either knotted or unknotted. In the same paper in which they proved K_6 is intrinsically linked, Gordon and Conway also proved that if the graph K_7 is embedded in space in any manner whatsoever ,it will always contain a Hamiltonian cycle that is knotted.

To find an embedding of K_7 containing no trefoil knots: First we need to look at a new invariant for knots and links called the Arf Invariant. Like the variable V we define in the earlier sections, the arf invariant will always have a value 0 or 1. There are several ways to define the arf invariant. We take a point of view due to Louis Kauffman. Let's define a pass-move to be a change in a projection as in the figure.



A pair of oppositely oriented strands can be passed through another pair of oppositely oriented strands. Such a move certainly can change the knot that we are dealing with. We call two knots pass equivalent if there exists a sequence of pass-moves that takes us from the one knot to the other, where we can rearrange the projection of the knot anyway that we want after each pass-move.

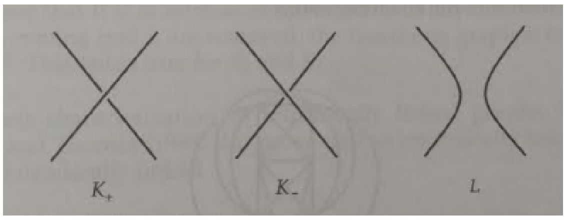
Every knot is pass equivalent to a composition of trivial knots and trefoil knots. However, since the composition of any knot K with the trivial knot just gives the knot K back again, we have shown that every knot is pass equivalent to either the trivial knots or a composition of trefoil knots. Left hand trefoil, appearing in the particular projection shown here, we can obtain its mirror image by passing all of the overlapping bands through each other.



We will now define the arf invariant a(K) of a knot K to be 0 if the knot is pass equivalent to the unknot and to be 1, if the pass equivalent to the trefoil knot.

The arf invariant has one very nice property, namely if K_+, K_- and L are projections that are identical outside the region shown, and if K_+ and K_- are knots, while L is a two-component link where each of the strands shown in the picture of L corresponds to a distinct component, then the arf invariants of the two knots are related through equation

$a(K_+) = a(K_-) + lK\ (L_1 ,L_2)$



Every embedding of K_7 contains a knotted Hamiltonian cycle. Given a particular embedding of K_7 we first define ω to be the sum of the Arf invariants summing over every Hamiltonian cycle in the graph. We actually don't care about ω itself, but rather, we care about whether it is odd or even. Therefore we define Ω to be 0 if ω is even and to be 1 if ω is odd. Conway and Gordon prove that a crossing change leaves Ω unaffected. Since Ω is unaffected by crossing changes, Ω must be the same for every embedding of K_7 . in particular if Ω =1 for any specific embedding, Ω =1 for every specific embedding. In fact it is tedious but not difficult to show that for the embedding of K_7 all of the Hamiltonian cycles except one are unknotted and the last Hamiltonian cycle is a trefoil knot. Hence Ω =1 for this embedding and therefore for all embedding. Finally, if Ω = 1, for every embedding, then for a given embedding, it cannot be the case that all the Hamiltonian cycles in that embedding are unknotted. Therefore every embedding of K_7 contains a knotted Hamiltonian cycle.

In 1988, Miki Shimabara proved that any embedding of the graph $K_{5,5}$ also contains a knotted Hamiltonian cycle. The graph $K_{5,5}$ is called bipartite graphs. It is obtained by taking two sets of 5 vertices and attaching each vertex in the first set to every one of the vertices in the second set by edges.

We say that a graph is intrinsically knotted if every embedding of the graph in three-space contains a knotted cycle. Note that if a graph contains a sub graph that is intrinsically knotted, it's also must be intrinsically knotted.

APPLICATIONS

Knots Theory in Chemistry

The Molecular chirality

One of the most important characteristics of a knot is its chirality. During the years, all the knots theorists have tried to find a way to determinate it. A molecule is said to be chemically achiral if it can be changed into its mirror image. Otherwise is said to be chemically chiral. This is the definition given in chemistry. In mathematics there are other two definitions, geometrical chirality and topological chirality according to the characteristics of the molecule, respectively rigid or flexible.

Establishing the topological chirality of a molecule

The following mathematical methods are used to find out if a molecule is topological chiral or not.

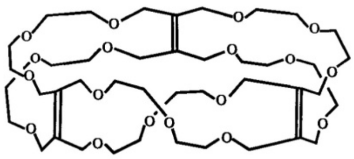


Figure3.12:AmolecularM¨obiusladder.

Method 1: Knot polynomial . This method can be used when a molecule is knotted. Different theorists tried to find ways to distinguish whether a knot is chiral or not. The Jones polynomial is the only method which can actually determinate such difference. Indeed every chiral knot and its mirror image have different Jones polynomials. However, if the Jones polynomial is the same it doesn’t necessarily mean that the knot is achiral. That is, the

Jones polynomial is useful toestablish topological chirality, but not for proving topological achirality. The onon disadvantage of this method is that not all the molecules are knotted. For example the Mo¨biu ladder (fig.3.12) doesn't contain any link or knot. So another method is needed in order to establish if this molecule is topologically chiral.

Knot theory in molecular Biology

F.H.C Crick and J.D Watson is one of the most remarkable insights of the 20th century ,unraveled the basic structure of DNA. A molecule of DNA may be thought of as two linear strands interwined in the form of a double helix with a linear axis . A molecule of DNA may also take the form of a ring ,and so it can become tangled or knotted .As DNA has the structure of two linear strands ,this however is the not only possible structure of DNA and in what follows the next description is probably more easy to comprehend in the context of knot theory the information the DNA molecule carries that is the arrangement of it's nucleotide base pair ,is unrelated to how it is knotted .so may be we should dismiss the knot as useful tool in molecular biology (without much significance). How ever recent researches has shown that the knot type has an important effect on the actual function of the DNA molecule in the cell. Therefore using knot theory techniques ,it may be possible to bring further insight into the structure of DNA molecule .At present the extent knot theory may further help in the understanding of mechanism of recombination of the DNA molecule .

CONCLUSION

In the project, about knots, their compositions and also the verities of knots have been reviewed. We also discussed about the links and graphs. We have seen that knot theory has many applications as in DNA synthesis. The original motivation for the founders of knot theory was to create a table of knot and links, which are knots of several components entangled each other. More than six billion knots and links are tabulated. Since the beginning of knot theory is in the nineteenth century. As the knot theory have been developing and the importance is also increased. The basic knot theory has numerous applications and we are still trying to work on the knot theory in higher dimensions.

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