INTRODUCTION TO TOPOLOGY

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CERTIFICATE

This is to certify that the project report titled "INTRODUCTION TO TOPOLOGY" submitted by SREELAKSHMI G(Reg no. 170021032434), SHAMIL JOLLY(Reg no. 170021032429) and AMAL T S(Reg. no:170021032394) towards partial fulfilment of the requirements for the award of Degree of Bachelor of Science in Mathematics is a bonafide work carried out by them during the academic year 2017-2020.

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DECLARATION

We ,SREELAKSHMI G (Reg. no:170021032434), SHAMIL JOLLY (Reg. no:170021032429) and AMAL T S(Reg. no:170021032394) hereby declare that this project entitled "INTRODUCTION TO TOPOLOGY" is an original work done by us under the supervision and guidance of Assistant prof. Nisha V M, faculty, Department of Mathematics in St. Paul's college Kalamassery in partial fulfilment for the award of The Degree of Bachelor of Science in Mathematics under Mahatma Gandhi University. We further declare that this project is not partly or wholly submitted for any other purpose and the data included in the project is collected from various sources and are true to the best of our knowledge.

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INTRODUCTION TO TOPOLOGY

Submitted by,

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CONTENT

INTRODUCTION

Topology is literally means the study of surfaces and is concerned with the property of a geometric object that are preserved under continuous deformation such as twisting bending but not tearing. It can be also used by persons from other areas of mathematics, computer science, physics etc. In this project, we aim to study the relation of geometry and topology. Further, we intended to study the various concepts on a topological spaces.

CHAPTER 1

PRELIMINARIES

SECTION-1.1

SETS AND FUNCTIONS

- *Axiomatic Set Theory*

  The approach to set theory in which an attempt is made to define a set an postulate a number of axioms about sets so as to avoid paradoxes is known as the axiomatic set theory.

- *Universal And Null Sets*

  All sets under investigation are subsets of a fixed set. We call this set the universal set or universe of discourse and denote by U. It is also convenient to introduce the concept of the empty or null set, that is, is a set which contains no elements. This set, denoted Φ is considered finite and a subset of every other set. Thus, for any set A,$\Phi \subset A \subset U$.

- *Empty Set*

It is possible to conceive a set with no elements at all. Such a set is variously known as an empty set

.

- *Subsets, Supersets*

A set A is a subset of a set B or, equivalently, B is a super set of A, written

$$A \subset B \text{ or } B \supset A$$

iff each elements in A also belongs to B; that is, if x ∈ A such that x ∉ B.

- *Proper Subset*

If S⊂T but S≠T, then we say that S is a proper subset of T.

- *Power Set*

If S is a set then the set of all subset of S is called the power set of S and will be denoted by P(S).

- *Pairwise Disjoint*

If A, B are two sets, we say that A is disjoin from B if $A \cap B$ is a empty set. Otherwise we say that A intersects B or that A and B intersects. A family T of set is said to be pairwise disjoint if every two distinct members of it are mutually disjoint.

- *Cartesian Product*

Let A, B be any sets. Then their Cartesian product is defined to be the set $\{(x, y): x \in A, y \in B\}$. It is denoted by $A \times B$.

- *Factor set*

The set whose Cartesian product is formed are called factor set or the factors.

- *Composition*

If $f: X \to Y$, $g: Y \to Z$ are functions, their composition or composite is denoted $g \circ f$ and is defined to be the function from X to Z given by $g \circ f(x) = g(f(x))$ for $x \in X$.

- *Constant Functions*

The simplest functions are the so called constant functions.

- *Injection, Surjection& Bijection*.

A function f: X→Y is said to be injective if for all $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

A function X→Y is said to be surjective if for each y ∈ Y there is some x∈ X such that f(x)=y.

A function which is both injective and surjective is called a bijective function or a bijection. A bijection of a set onto itself is called a permutation of that set.

- *Denumerable*

A set X is finite if either X=∅ or there exists a bijection f: {1, 2 ,………, n}→X for some positive integer n. Otherwise it is infinite. It is denumerable if there exists a bijection f: N→X where N is the set of all positive integers. A set which is either finite or denumerable is said to be countable, otherwise it is uncountable.

- *Equipollent*

Two sets X and Y are said to be equipollent to each other if there exist a bijection f: X→Y. It is easy to show that if X, Y are equipollent and Y, Z are equipollent then X, Z are equipollent.

SECTION-1.2

SETS WITH ADDITIONAL STRUCTURE

- *Binary Relation*

If S is a set, a binary relation on S is defined simply as a subset of S×S.

- *Reflexive, Symmetric & Transitive*

A relation R on asset S is said to be reflexive if for all a ∈ S, aRa.

It is symmetric if for all a, b ∈ S, aRb implies bRa.

It is transitive if for all a, b, c ∈S, aRb and bRc implies aRc. It is antisymmetric if for all a, b ∈S, aRb and bRa implies a=b.

- *Equivalence Relation*

A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive. If R is an equivalence relation on S and $x \in S$ then R[x] is called the equivalence class under R or R-equivalence class containing x.

- *Partial Order*

A relation which is reflexive, transitive and antisymmetric is called a partial order.

- *Order Preserving*

If $(S, \leq_1)$ and $(T, \leq 2)$ are two partially ordered sets and $f: S \rightarrow T$ is a function then f is said to be order preserving.

- *Order Isomorphism*

I f is a bijection and f as well as its inverse f^{-1} are both order preserving then f is called an order isomorphism.

- *Bounded Above and Bounded Below*

An element $x \in S$ is said to be an upperbound for A if for all $a \in A$, $a \leq x$. A set which has at least one upperbound is said to be bounded above. The terms 'lower bound' and 'bounded below' are defined similarly. A set which is both bounded above and bounded below is said to b bounded.

- *Supremum and Infimum*

The least element, if any, of the set of all upper bounds of asset A is called the least upper bound of A and is denoted by sup(A) or by sup A.

The concept of the greatest lower bound of a set A is defined analogously and is denoted by inf (A) or by inf A.

- *Ring*

(R, +) is an abelian group and • is another binary operation on R which is associative and distributive over + then the triple (R, +, •) is called a ring.

- *Open set*

Let X be a metric spaces with metric d. If x_0 is a point of X and r is a positive real number open sphere $S_r(x_0)$ with centre x_0 and radius 'r' is the subset of X is defined by $S_r(x_0)=\{x/d(x,x_0)<r\}$.

CHAPTER 2
MOTIVATION FOR TOPOLOGY

SECTION- 2.1

TOPOLOGY

The word 'Topology' is derived from two Greek words, topos meaning 'surface' and logos meaning 'discourse' or 'study'. Topology thus literally means the study of surfaces. Topology studies properties of spaces that are invariant under any continuous deformation. It is sometimes called "rubber-sheet geometry" because the objects can be stretched and contracted like rubber, but cannot be broken. For example, a square can be deformed into a circle without breaking it. Topology is a relatively new branch of mathematics; most of the research in topology has been done since 1900. The following are some of the subfields of topology.

1. ***General Topolgy or Point Set Topology*:** General topology normally considers local properties of spaces, and is closely related to analysis. It generalizes the concept of continuity to define topological spaces, in which limits of sequences can be considered. Sometimes distance can be defined in these spaces, in which case they are called

metric spaces; sometime no concept of distance makes sense.

2. *Combinatorial Topology*:

Combinatorial topology considers the global properties of spaces, built up from a network of vertices, edges, and faces. This is the oldest branch of topology, and dates back to Euler. It has been shown that topologically equivalent spaces have the same numerical invariant, which we now call the Euler characteristic. This is the number (V-E+F), where V, E and F are the number of vertices, edges, and faces f an object. For example, a tetrahedron and a cube are topologically equivalent to a sphere, and any '"triangulation'" of a sphere will have an Euler characteristic of 2.

3. *Algebraic Topology*:

Algebraic topology also considers the global properties of spaces, and uses algebraic objects such as groups and rings to answer topological questions. Algebraic topology converts a topological problem into an algebraic problem that is hopefully easier to solve. For an algebraic problem that is hopefully easier to solve. For example, a group called a homology group can be associated to each space, and the torus and the Klein bottle can be distinguished from each other because they have different homology groups.

Algebraic topology sometimes uses the combinatorial structure of a space to calculate the various groups associated to that space.

4. ***Differential Topology*:** Differential topology considers spaces with some king of smoothness associated to each point. In this case, the square and the circle would not be smoothly (or differentiably) equivalent to each other, Differential topology is useful for studying properties of vector fields, such as a magnetic or electric fields.

Topology is used in many branches of mathematics, such as differentiable equations, dynamical systems, knot theory, and Riemann surfaces in complex analysis, it is also used in string theory in physics, and for describing the space-time structure of universe.

GEOMETRY AND TOPOLOGY

We remarked that topology, like geometry, deals with certain 'object', classifies them according to some equivalence relation and then studies those properties of the objects which are invariant under this classification. *Geometry* has local structure while *topology* only has global structure. Alternatively, geometry has continuous *moduli*, while topology has discrete moduli. The study of metric spaces is geometry, the study of topological spaces is topology.

Definition: Let A, B be subsets of Euclidean spaces f: A→B a function and $x_0 \in$ A. We say f is continuous at x_0 if for each $\in > 0$, there exists $\delta > 0$ such that $d(f(x)),f(x_0) < \in$ for all $x \in A$ for which $d(x,x_0) < \delta$; where as usual, the same symbol d is used to denote the distance between point of A as well as B. Further, we say f is continuous, if it is **continuous** at all point of A.

Definition: Let A, B be subsets of Euclidean spaces. A homeomorphism from A to B is a bijection f: A→B such that both f and its inverse are are continuous. When such f exists, A and B are said to be **homeomorphic** to each other.

Moduli

If a structure has a discrete moduli, the structure is said to be rigid, and its study is topology. If it has non-trivial deformations, the structure is said to be flexible, and its study is geometry.

The space of homotopy classes of maps is discrete, so studying maps up to homotopy is topology. Similarly, differential structures on a manifold is usually a discrete space.

Symplectic Manifolds

Symplectic manifolds are a boundary case, and parts of their study are called symplectic topology and sympletic geometry.

By Darboux's theorem, a symplectic manifolds has no local structure, which suggests that their study be called topology.

The space of symplectic structures on a manifolds form a continuous moduli, which suggests that their study be called geometry.

CHAPTER 3
TOPOLOGICAL SPACES

SECTION-3.1

TOPOLOGICAL SPACES

Let X be non-empty set. A class *T* of subsets of X is a topology on X iff *T* satisfies the following axioms.

[O_1] X and ∅ belong to *T.*

[O_2] The union of any number of sets in *T* belongs to *T.*

[O_3] The intersection of any two sets in *T* belongs to *T.*

The members of *T* are then called *T*-open sets, or simply open sets, and X together with *T*, i.e. the pair (X,*T*) is called a topological space.

Definitions:

1. Then *u* is a topology on R; it is called the *usual topology* on R. Similarly, the class. Similarly, the class *u* of all open sets in the plane R^2 is a topology, and also called the *usual topology* on R^2.
2. Let *D* denote the class of all subsets of X. *D* satisfies the axioms for a topology on x. This topology is called the discrete topology; and X together with its discrete topology, i.e. the pair (X, *D*), is called a *discrete topological spaces*.
3. As seen by axioms [O_1], a topology on X must contain the sets X and ∅. The class g={X,∅}, consisting of X and ∅ alone, is itself a topology on

X. It is called the indiscrete topology; and X together with its indiscrete topology, i.e. (X, g), is called an *indiscrete topological space*.

4. Let T denote the class of all subsets of X whose complements are finite together with the empty set ∅. This class T is also a topology on X. It is called the *cofinite topology* or the T_1- topology on X.

5. If G is an open set containing a point $p \in X$, then G is called an *open neighborhood* of p. Also, G without p, i.e. $G \setminus \{p\}$, is called a *deleted open neighborhood* of p.

Open Ball

Let $x_0 \in X$ and r be a positive real number. Then the open ball with centre x_0 and radius r is defined to be the set $\{x \in X: d(x, x_0) < r\}$. It is denoted either by $Br(x_0)$ or by $B(x_0; r)$. It is called the open r-ball around x_0.

Proposition: Let $\{x_n\}$ be a sequence in a metric space (X; d). Then $\{x_n\}$ converges to y in X iff for every open set U containing y, there exists a positive integer N such that for every integer $n \geq N$, $xn \in U$.

Let f: X→Y be a function where X, Y are metric spaces and let $x_0 \in X$. then f is continuous at x_0 iff for every open set V in Y containing $f(x_0)$, there exists an open set U in x containing x_0 such that $f(U) \subset V$.

Theorem:

Let (X, d) be a metric space. Then

1. The empty set ∅ and the entire set x are open.
2. The union of any family of open sets is open.
3. The intersection of any finite number of open set is open.
4. Given distinct point x, $y \in X$ there exist open sets U, V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Proof

1 and 2 are trivial consequence of the definition of open sets.

For 3 first consider the case of the intersection of two open sets say A_1 and A_2. Let $x \in A_1 \cap A_2$. Then $x \in A_1$ and $x \in A_2$. Since A_1 is open, there exists $r_1 > 0$ such that $B(x;r_1) \subset A_1$. Similarly since A_2 is open there exists $r_2 > 0$

such that $B(x;r_2) \subset A_2$. Now let $r = \min\{r_1, r_2\}$. Then clearly $B(x;r) \subset B(x:r_1) \cap B(x:r_2) \subset A_1 \cap A_2$. Thus $A_1 \cap A_2$ is open. One can either generalize this argument or use induction to settle the general case. The exceptional case of the intersection of an empty family of open sets is already covered under 1.

For 4 let $x, y \in X$ and $x \neq y$. then $d(x,y) > 0$. Choose r so that $0 < r < d(x,y)/2$ and let $U = B(x,r)$, $V = B(y, r)$. then clearly U, V are open sets containing x, y respectively. Also they are mutually disjoint, for if $z \in U \cap V$ then $d(x,z) < r$ and $d(x,z) < r$ when $d(x,y) < 2r$ by triangle inequality, a contradiction.

Examples:

1. Let X= {a,b,c,d,e}. determine whether or not each of the following classes of subsets of X is a topology on X.
   i. $T_1 = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\}$
   ii. $T_2 = \{X, \emptyset, \{a,b,c\}, \{a,b,d\}, \{a,b,c,d\}\}$
   iii. $T_3 = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}\}$

Solution:

i. T_1 is not a topology on X since $\{a,b\}, \{a,c\} \in T_1$ but $\{a,b\} \cup \{a,c\} = \{a,b,c\} \notin T_1$.

ii. T_2 is not a topology on X since {a,b,c}, {a,b,d} ∈ T_2 but {a,b,c}∩{a,b,d}= {a,b}∉ T_2.

iii. T_3 is a topology on X since it satisfies the necessary axioms.

2. Let T be a topology on asset X consisting of four sets, i.e. T= {X,∅,A,B} where A and B are non empty distinct proper subsets of X. What conditions must A and B satisfy?

Solution:

Since A∩B must also belong to T, there are two possibilities:

Case 1. A∩B=∅

Then A∪B cannot be A or B; hence A∪B= X. Thus the class {A,B} is a partition of X.

Case 2. A∩B= A or A∩B= B

In either case, one of the sets is a subset of the other, and the member of T are totally ordered by inclusion: ∅⊂A⊂X or ∅⊂B⊂A⊂X.

SECTION-3.2

BASES AND SUB-BASES

Base For A Topology

Let (X,T) be a topological space. A class B of open subsets of X, i.e. B⊂T, is a base for the topology T iff

i. Every open set G ∈ T is the union of members of B. Equivalently, B⊂T is a base for T iff
ii. For any point p belonging to an open set G, there exists B∈B with p∈B⊂G.

Example:

1. Let B be a base for a topology T on X and let B* be a class of open sets containing B, i.e. B⊂B*⊂ T. Show that B* is also a base for T.

Solution:

Let G be an open subset of X. since B is a base for (X, T), G is the union of members of B, i.e. G= $\cup_i B_i$ where B_i ∈ B. But B⊂ B*; hence each Bi ∈B also belongs to B*. So G is the union of members of B*, and therefore B* is also a base for (X, T).

2. Let B and B* be bases, respectively, for topologies T and T* on a set X. Suppose that each B∈ B is the union

of members of B*. Show that *T* is coarser than *T**, i.e. $T \subset T^*$.

Solution:

Let G be a *T*-open set. Then G is the union of members of B, i.e. $G= \cup_i B_i$ where $B_i \in B$. But, by hypothesis, each $B_i \in B$ is the union of members of B*, and so $G=\cup_i B_i$ is also the union of members of B* which are *T**-open sets. Hence G is also a *T**-open set, and so $T \subset T^*$.

SUB-BASES

Let (X, *T*) be topological space. A class S of open subsets of X, i.e. $S \subset T$, is a sub-base for the topology *T* on X iff finite intersections of members of S form a base for *T*.

Theorem:

Let X be a set, *T* a topology on X and S a family of subsets of X. Then S is a sub-base for *T* iff S generates *T*.

Proof:

Let B be the family of the finite intersections of members of S. Suppose first that S is a sub-base for T. We want to show that T is the smallest topology on X containing S. Now since S⊂B and B⊂ T we at least have that T contains S. Suppose U is on another topology on X such that S⊂U. We have to show that T⊂U. Now U is closed under finite intersections and S⊂U, U contains all finite intersection of members of S, i.e. B⊂U. But again since U is closed under arbitrary union and each members of T can be written as union of some members of B, it follows that T⊂U.

Conversely suppose T is the smallest topology containing S. We have to show that S is a sub-base for T, i.e. that B is a base for T. Clearly B⊂ T since T is closed under finite intersection and S⊂ T. Since B is closed under intersection, there is a topology U on X such that B is a base for u. Every member of U can be expressed as a union of a sub-family of B and so is in T since B⊂ T. This means U⊂ T and consequently U= T since T is the smallest topology containing S. Thus B is a base for T S is a base for T.

Examples:

1. Show that all intervals (a,1] and [0,b), where $0<a,b<1$, form a sub-base for the relative usual topology on the unit interval I=[0,1].

Solution:

Recall that the infinite open intervals (a,∞) and (-∞) form a sub-base for the usual topology on the real line R. The intersection of these infinite open intervals with I=[0,1] are the sets ∅,I,(a,1] and [0,b] which, by the preceding problem, form a sub-base for I=[0,1]. But we can exclude the empty set ∅ and the whole space I from any sub-base; so the intervals (a,1) and [0,b) form a sub-base for I.

2. Show that if S is a sub-base for topologies T and T^* on X, then $T = T^*$.

Solution:

Suppose $G \in T$. Since S is a sub-base for T, $G = \cup_I (S_1 \cap \ldots\ldots \cap S_n)$, where $S_k \in S$.

But S is also a sub-base for T^* and so $S \subset T^*$; hence each $S_k \in T^*$. Since T^* is a topology, $S1 \cap \ldots\ldots \cap S_n \in T^*$ and hence $G \in T^*$. Thus $T \subset T^*$. Similarly $T^* \subset T$, and so $T = T^*$.

SECTION-3.3

SUBSPACES, RELTIVE TOPOLOGIES

Let A be a non empty subset of a topological space (X, $\mathcal{T}$). The class $\mathcal{T}_A$ of all intersections of A with $\mathcal{T}$-open subsets of x is a topology on A; it is called the relative topology on A or the relativization of $\mathcal{T}$ to A, and the topological space (A, $\mathcal{T}_A$) is called a subspaces of (X, $\mathcal{T}$). In other words, a subset H of A is a $\mathcal{T}_A$-open set, i.e. open relative to A, if and only if there exists a $\mathcal{T}$-open subset G of X such that

$$H=G\cap A$$

Example:

1. Consider the following topology on X={a, b, c, d, e}:

   $\mathcal{T}$={X,∅,{a},{a,b},{a,c,d},{a,b,c,d},{a,b,e}}

   List the members of the relative topology $\mathcal{T}_A$ on

   A= {a, c, e}.

Solution:

$\mathcal{T}_A$= {A∩G: G∈ $\mathcal{T}$}, so the members of $\mathcal{T}_A$ are:

A∩ X=A A∩{a}={a} A∩{a,c,d}={a,c}

A∩{a,b,e}={a,e} A∩ ∅=∅ A∩{a,b}={a}

A∩{a,b,c,d}={a,c}

In other words, $\mathcal{T}_A$ = {A,∅,{a},{a,c},{a,e}}. Observe that {a, c} is not open in X, but is relatively open in A, i.e. is $\mathcal{T}_A$-open.

2. Let A be a $\mathcal{T}$- open subser of (X, $\mathcal{T}$) and let $A \subset Y \subset x$. Show that A is also open relative to the relative topology on Y, i.e. A is a $\mathcal{T}_Y$-open subset of Y.

Solution:

$\mathcal{T}_Y = \{Y \cap G : G \in \mathcal{T}\}$. But $A \subset Y$ and $A \in \mathcal{T}$; so $A = Y \cap A \in \mathcal{T}_Y$.

CHAPTER 4

BASIC CONCEPTS

Neighborhoods

Let (X; T) be a topological space, $x_0 \in X$ and $N \subset X$. Then N is said to be a neighborhood of x_0 or x_0 is said to be an interior point of N if there is an open set V such that $x_0 \in V$ and $V \subset N$.

Theorem:

A subset of a topological space is open iff it is a neighborhood of each of its points.

Proof:

Let X be a topological space and $G \subset X$. First suppose G is open. Then evidently G is a neighborhood of each of its points. Conversely suppose G is a neighborhood of each of its points. Then for each $x \in G$, there is an open set V_x such that $x \in V_x$ and $V_x \subset G$. Since each V_X is open so is G.

Trivially if a neighborhood of a point x then so is any superset of N. It is also easy to show that the intersection of any two neighborhoods of a point is a again a neighborhood of that point.

Interior

Let (X, T) be a space and $A \subset X$. Then the interior of A is defined to be the set of all interior points of A, i.e. the set {x∈ A: A is a neighborhood of x}. It is denoted by A^0 or int(A).

Example:

Consider the following topology on X= {a, b, c, d, e }:

$\mathcal{T}$={X,∅,{a},{a,b},{a,c,d},{a,b,c,d},{a,b,e}}

Find the interior point of the subset A={a,b,c} of X.

Solution:

The point a and b are interior point of A since a, b ∈ {a, b} ⊂ A= {a, b, c}, where {a,b} is an open set, i.e. since each belongs to an open set contained in A. Note that c is not an interior point of a since c does not belong to any open set contained in A. Hence int(a)= {a,b} is the interior of A.

Accumulation Points

Let A be a subset of a topological space X and y∈ X. Then y is said to be an accumulation point of A if every open set containing y contains at least one point of A other than y.

Derived set

Let A be a subset of a space X. Then the derived set of A, denoted by A', is the set of all accumulation points of A in X.

Theorem:

For a subset A of a space X, $\bar{A} = A \cup A'$.

Proof:

First we claim that $A \cup A'$ is closed or that $X-(A \cup A')$ is open. We do so by showing that $X-(A \cup A')$ is a neighborhood of each of its points. Let $y \in X-(A \cup A')$. Then since y is not a point of accumulation of A there exists and open set V containing y such that V contains no point of A except possibly y. But $y \notin A$, so we have $A \cap V = \emptyset$. We claim $A' \cap V$ is also empty. For, let $z \in A' \cap V$. Then V is an open set containing z which is an accumulation point of A. So $V \cap A$ is nonempty, a contradiction. So $A' \cap V = \emptyset$ and hence $V \subset X-(A \cup A')$. This proves that $A \cup A'$ is closed and since it obviously contains A, it also contains $\bar{A}$; i.e. $\bar{A} \subset A \cup A'$.

For the other way inclusion, $A \cup A' \subset \bar{A}$, it suffices to show that $A' \subset \bar{A}$ since we already have $A \subset \bar{A}$. So let $y \in A'$. If $y \notin \bar{A}$ then $y \in X-\bar{A}$ which is an open set since $\bar{A}$ is always a closed set. But y is an accumulation point of A. So $(X-A) \cap A \neq \emptyset$ which is a contradiction since $X-\bar{A} \subset X-A$. So $y \in \bar{A}$. Hence the proof.

CONCLUSION

Topology is an area of Mathematics concerned with the properties of space that are preserved under continuous deformations including stretching and bending, but not tearing. By the middle of the 20^{th} century, topology had become a major branch of mathematics.

Topology as a branch of mathematics can be formally defined as the study of qualitative properties of certain objects that are invariant under a certain kind of equivalence and it s the study of those properties of geometric configurations which remain when these configurations are subjected to one-to-one bicontinuous transformations or homeomorphisms. Topology operates with more general concepts than analysis. Differential properties of a given transformation are nonessential for topology but bicontinuity is essential. As a consequence, topology is often suitable for the solution of problems to which analysis cannot give the answer.

Topology is used in several areas such as quantum field theory, image processing, molecular biology and cosmology and can also be used to describe the overall shape of the universe. The various possible positions of a robot can be described by a manifold called configuration space. In the area of motion planning one finds paths between two points in configuration space.

General topology is important in many field of applied science as well as branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computeraided design, digital topology, information systems, particle physics and quantum physics etc.

The notations of sets and functions in topological spaces, ideal topological spaces and ideal minimal spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences.

By researching generalizations of closed sets in various fields in general topology, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, all functions defined in this thesis will have many possibilities of applications in digital topology and computer graphics.

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