FRACTAL GEOMETRY

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DEPARTMENT OF MATHEMATICS

ST. PAUL'S COLLEGE, KALAMASSERY

(AFFILIATED TO M G UNIVERSITY, KOTTAYAM)

CERTIFICATE

This is to certify that the project report titled "**FRACTAL GEOMETRY**" submitted by **AN MARIYA JOY**(Reg no. 17002103243595), **SHILPA I.S**(Reg no. 170021032431) and **SOORYA SABU**(Reg. no:170021032433) towards partial fulfilment of the requirements for the award of Degree of Bachelor of Science in Mathematics is a bonafide work carried out by them during the academic year 2017-2020.

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DECLARATION

We , **AN MARIYA JOY** (Reg. no:170021032395), **SHILPA I.S**(Reg. no:170021032431) and **SOORYA SABU**(Reg. no:170021032433) hereby declare that this project entitled "FRACTAL GEOMETRY" is an original work done by us under the supervision and guidance of Ms. Maya K, faculty, Department of Mathematics in St. Paul's college Kalamassery in partial fulfilment for the award of The Degree of Bachelor of Science in Mathematics under Mahatma Gandhi University. We further declare that this project is not partly or wholly submitted for any other purpose and the data included in the project is collected from various sources and are true to the best of our knowledge.

Place: Kalamassery

Date:

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For any accomplishment or achievement, the prime requisite is the blessing of the Almighty and it's the same that made this world possible. We bow to the lord with a grateful heart and prayerful mind.

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ABSTRACT

Fractals is a new branch of mathematics andart. Perhaps this is the reason why mostpeople recognize fractals only as prettypictures useful as backgrounds on the computer screen or original postcardpatterns. But what are they really?

Most physical systems of nature and manyhuman artifacts are not regular geometricshapes of the standard geometry derivedfrom Euclid. Fractal geometry offers almostunlimited waysof describing, measuring andpredicting these natural phenomena. But is itpossible to define the whole world usingmathematical equations ?

Basic concepts,methods and applications are discussed in this project.

CONTENT

CHAPTER 1

INTODUCTION

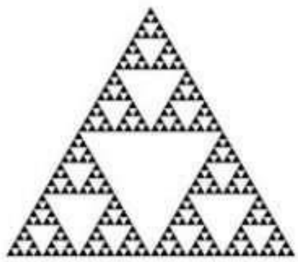
Although the term “modern geometries” traditionally refers to post-Euclid geometries, namely the non-Euclidean and projective geometries,it seems ironic to describe topics presented in this, on the other hand, are among those in a newly emerging area of mathematics and are honestly modern. In fact, the area known as fractal geometry is so new that its exact content has yet to be determined, let alone given a formal axiomatic structure. Thus, this contains an informal presentation of concepts and themes basic to the topics currently regarded as part of fractal geometry. This presentation also attempts to convey the excitement experienced by professional mathematicians and scientists as they discover and comprehend these new ideas and contemplate their far reaching applications

A fractal is a never-ending pattern. Fractals are infinitely complex patterns that are selfsimilar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop. Driven by recursion, fractals are images of dynamic systems – the pictures of chaos. Geometrically, they exist in between our familiar dimensions . Fractal patterns are extremely familiar, since nature is full of fractals.For instance : trees, rivers, coastlines, mountains, clouds, seashells, hurricanes,etc . Abstract fractals such as the Mandelbrot set can be generated by a computer calculating a simple equation over and over.

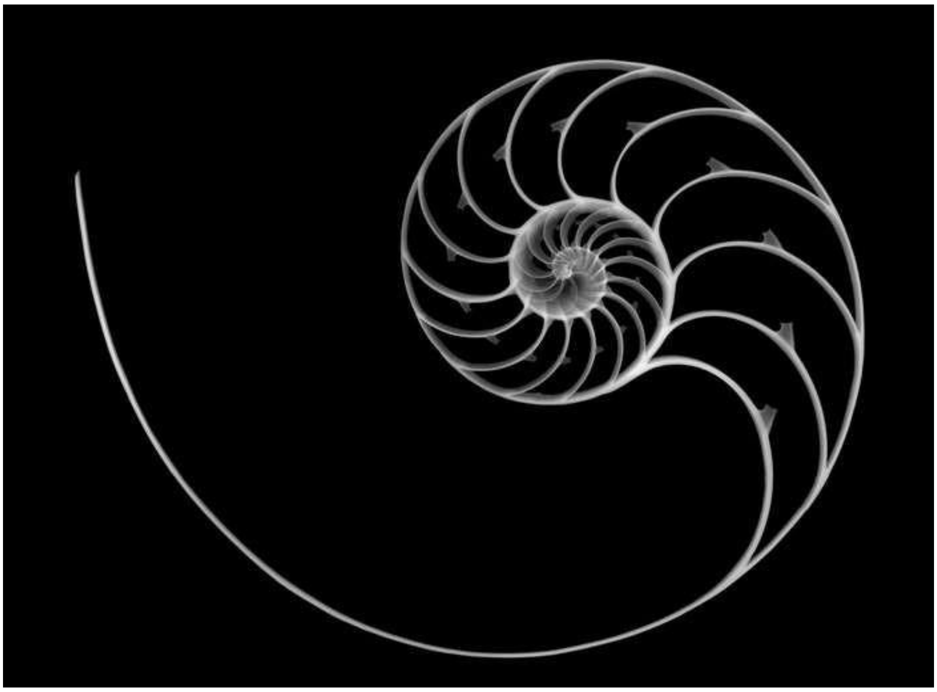
Fractals are not just complex shapes and pretty pictures generated by computers. Anything that appears random and irregular can be fractal. Fractals are used to model soil erosion and to analyze seismic patterns as well. Seeing that so many facts of mother nature exhibit fractal properties, maybe the whole world is a fractal afterall ! Fractals permeate our lives, appearing in place as tiny as the membrane of a cell and as majestic as the solar system. Fractals are the unique, irregular patterns left behind by the unpredictable movements of the chaotic world at work. Actually, the most useful use of fractals in computer science is the fractal image compression.

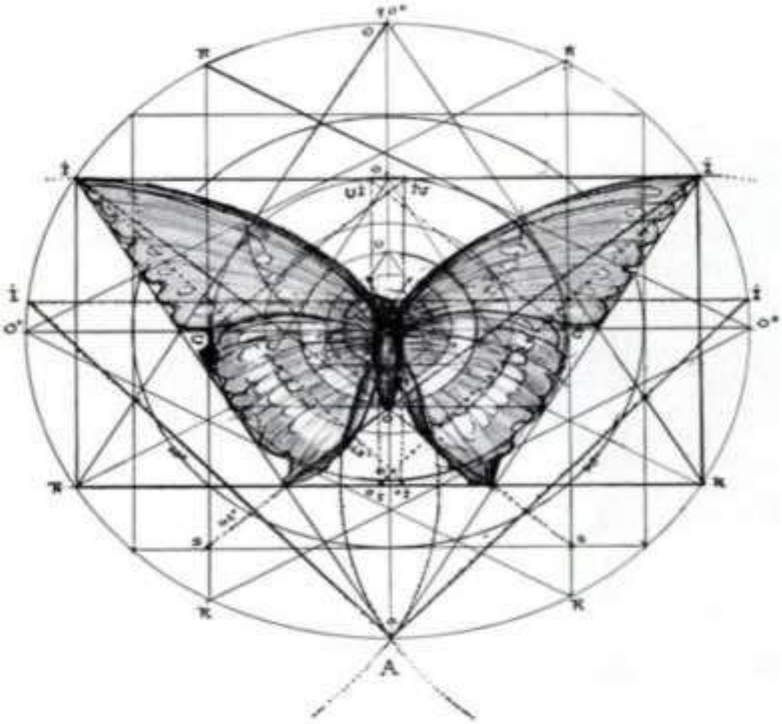
In nature:

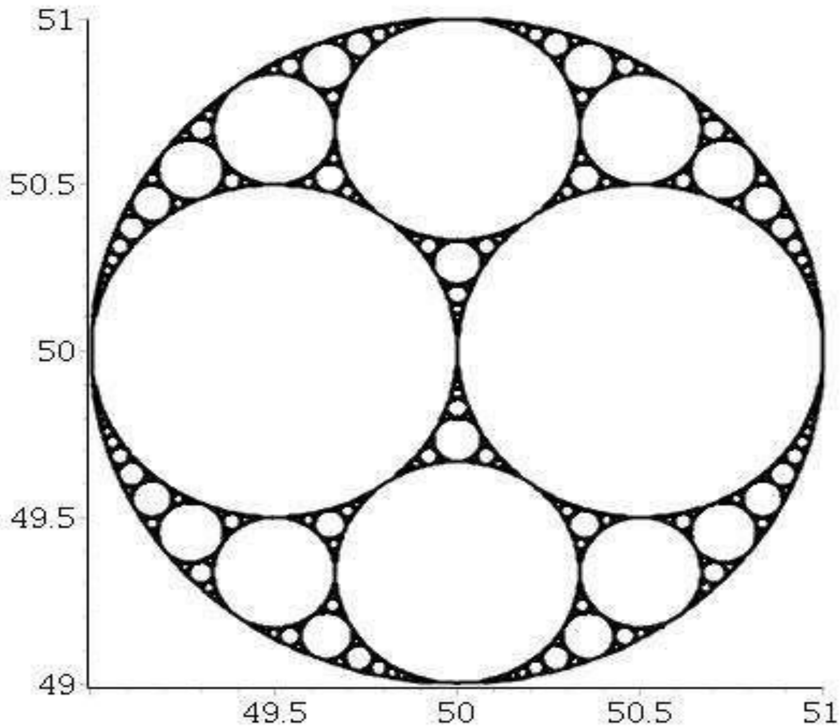
In geometry:



In algebra:



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CHAPTER 2

CHAOS

2.1- A CHAOTIC BACKGROUND

A mathematical definition of chaos would say something like deterministic behaviour that appears to be random".

To understand the significance of fractals and their role in modern mathematics, it is necessary to know something about the area of scientific and mathematical enquiry known as chaos theory. The importance of this new area, together with the excitement and frustrations experienced by scientists and the mathematicians as they made their initial discoveries , often without knowledge of other related work is wonderfully portrayed in James Gleick's chaos : Making a new science.

Gleick suggests that chaos is " a new science of the global nature of systems" and notes that some consider chaos to be the third great revolution in twentieth century science, placing it on the same level as relativity and quantum mechanics. Gleick continues : "As one physicist put it : ' Relativity eliminated the Newtonian illusion of absolute space and time ; quantum theory eliminated the Newtonian dream of a controllable measurement process ; and chaos eliminates the Laplacian fantasy of deterministic probability '.Of the three, the revolution in chaos applies to the universe we see and touch, to objects at human scale".

The attitude of science at the time initial discoveries in chaos were being made is summarised by Gleick as follows : As one theoretician liked to tell his students: " The basic idea of western science is that you don't have to take into account the falling of a leaf on some planet in another galaxy when you're trying to account for the motion of a billiard ball on a pool table on earth. Very small influences can be neglected. There's a convergence in the way things work, and arbitrarily small influences don't blow up to have arbitrarily large effects. Classically, the belief in approximation and convergence as well justified. It worked". However, as Gleick indicates, and as the quotation below confirms, chaos created a revolution in scientific thought.

The magnificent success in the fields of the natural sciences and technology had, for many, fed the illusion that the world on the whole functioned like a huge clockwork mechanism, whose laws were only waiting to be deciphered step by step. Once the laws were known, it was believed, the evolution or development of computer technology and its promises of a greater command of information, many have put increasing hope in these machines.

But today it is exactly those at the active core of modern science who are proclaiming that this hope is unjustified; the ability to see ever more accurately into future developments is unattainable. Once conclusion that can be drawn from the new theories is that stricter determinism and apparently accidental development are not mutually exclusive, but rather that their coexistence is more the rule in nature. Chaos theory and fractal geometry address this issue.

The term “Chaos” was introduced into the dynamical systems area in 1975 when researchers Li and Yorke published a paper titled “Period Three Implies Chaos” . Lorenz notes, “whatever they may have intended to do , they succeeded in establishing a new scientific term, although one with a somewhat different meaning from what they had in mind”.
Although chaos theory can be intuitively described as the theory of “complicated dynamical system”, so-called chaotic behaviour can occur even when functions as simple as quadratics are iterated, thus making basic concepts of chaos accessible at a reasonably elementary level . Indeed, Devaney notes “This is a fundamental break through made by mathematicians in recent years,the realization that chaotic systems need not depend on huge numbers of variables but may in fact depend on only one, as in the case of the family of logistic functions”.

The dynamic system in which Chaos arises are deterministic since they evolve according to precise rules given by equations which are usually not linear and often involve several variables. However, even when the equations of a system are not very complicated, the systems can be unstable due to their sensitive dependence on initial conditions.

As a result, the behaviour described by the system can become extremely complicated and, in the long run, unpredictable. And in cases like weather models,where we know the initial conditions somewhat imprecisely, very slight differences in the values of these initial conditions may lead to vastly different results. This leads to a dim future for long-range weather forecasting.

It is now realized that no new better simulation of weather on more accurate computers of the future will be able to predict the weather more than about fourteen days ahead, because of the very nonlinear nature of the evolution of the state of the weather.

Thus, the innocuous sounding butterfly effect provides a major counterexample to the classical assumption that arbitrarily small influences do not have arbitrarily large effects.

2.2- FAMOUS SETS IN CHAOS THEORY

Chaos theory is a part of mathematics. It looks as certain systems that are very sensitive. A very small change may make the system behave completely differently. The collection of sets that play significant role in chaos theory, namely Julia sets and the even more well known Mandelbrot set. Julia sets are named after the French mathematician Gaston Julia, who together with pierse Fatou, invented and studied these sets in the early 20^{th}century. The Mandelbrot set is named after the contemporary french mathematician Benoit Mandelbrot whose work from the 1950s through the 1970s at IBM In New York is generally recognised as the foundation of fractal geometry. With this connection, it is most appropriate that the Mandelbrot set has become a "logo" for fractal geometry and chaos theory. Its spectacular color representations generated by high speed modern computers are appreciated by mathematicians and nonmathematicians alike.

JULIA SETS

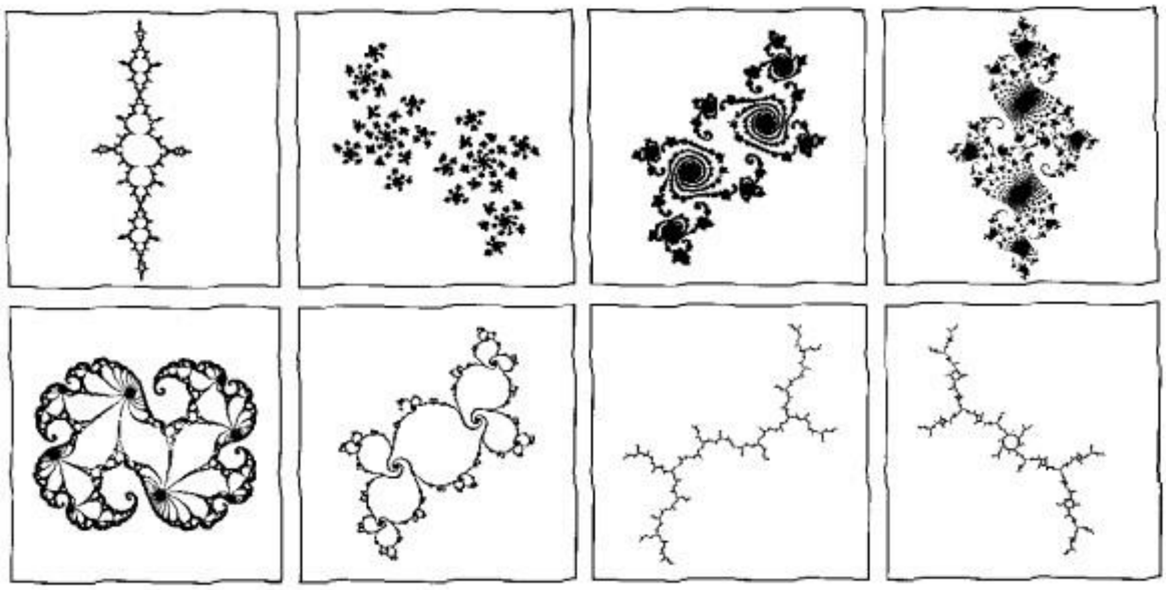
The way the graphical representation of a Julia set is obtained is distinctly different from the usual mathematical process of plotting a curve representing a function. For a function f of one real variable, the latter process, commonly known as "graphing the function", involves substituting a real number $x_1$for the variable x in the function expression; computing the result, $x_2=f(x_1)$; and plotting the point with coordinates(x_1,x_2)on a two-dimensional cartesian graph. However, to obtain a representation for the Julia set of a function f,an initial variable value, usually denoted x_0 , is substituted into the function expression yielding a new value $x_1=f(x_0)$. This new value is then used to produce a value $x_2=f(x_1)$and this process is iterated repeatedly to produce the orbit of x_0. This iteration continues until the long term orbit behaviour can be identified. Then the point x_0 is plotted in a color indicating this orbit behaviour. Generally, the points with bounded orbits are shown in black.

DEFINITION2.2.1

For a given function f, the set of points whose orbits are bounded under iteration of the function f is called the filled Julia set of f. The boundary of a filled Julia set is called the Julia set.

Logistic functions have associated Julia sets, but since the initial values used for logistic functions consist only of single-variable real numbers, the graphical representation of such Julia sets is merely a set of points on the x-axis , and hence rather uninteresting. To obtain two-dimensional Julia sets, we move into the realm of complex functions, that is, functions whose domain and range are sets of complex numbers. Here when an initial complex valuez_0 produces a bounded orbit, the point z_0 is plotted, that is, colored black, in the two dimensional complex plane. Some of the simplest functions that yield interesting Julia sets are the quadratic complex functions of the form $Q_c(z)=z^2+c$, where each complex constant c yields a different function in this family and hence a different shaped Julia set.

In the Julia set illustration given below, points of the complex plane shown in black have very predictable orbits, for example, the orbits either converge to specific points or cycle.In color illustrations, points depicted with colors have orbits that diverge to infinity. Different colours indicate different "speeds" with which the orbits diverge to infinity.



As definition indicates, the boundary between the set of black points and the set of colored(in our case white) points is the actual Julia set while the boundary and its interior are called the filled Julia set. For example, for the function$Q_0(z)=z^2$points z where $|z|<1$ all have orbits that converge to 0. Points z,where $|z|>1$ all have orbits that diverge to infinity. Thus, the Julia set of Q_0 is exactly the set of points on the circle $|z|=1$. However, points on this "simple" Julia set exhibit the same unpredictable behaviour that points on any Julia set exhibit under iteration by its defining function namely, any point on the circle $|z|=1$ has an orbit that does not "escape", that is, diverge,to infinity but the point itself is arbitrarily close to other points whose orbits under Q_0 eventually hit any nonzero point in the plane. Thus, just as in the case of Lorenz's weather model, there is sensitive dependence on initial conditions.

As might be expected from such complicated properties, Julia sets can be extremely intricate, and come in a vast array of shapes. One indication of the range of diversity in their shapes is the fact that some Julia sets are connected, that is, they are in one piece, while others are totally disconnected and thus described by the picturesque term “fractal dust”.

THE MANDELBROT SET

As complicated as Julia sets appear, there is another set that is more complicated and has been labelled “the most complex object in mathematics” . Mandelbrot discovered this set in the late 1970s when he attempted to make generalizations about Julia sets. Whereas individual members of the family of complex quadratic functions, $Q_c(z)=z^2+c$, each determine a Julia set in the z- plane, the Mandelbrot set is plotted in the c-plane and represents a catalog of the entire function family$\{Q_c(z)\}$. The characteristic property of Julia sets catalogued by the Mandelbrot set is that of connectedness- weather or not the Julia set of Q_c consists of exactly one piece.

DEFINITION 2.2.2

The Mandelbrot set is the set of complex numbers C such that the Julia set for the function $Q_c(z)=z^2+c$ is connected.

At first glance, this set appears to be nearly impossible to find, since determining whether a single complex point c is in the set would seem to require not only finding the entire Julia set for the function Q_c but also determining whether this Julia set is connected. Fortunately, there is a more efficient method of determination as a result of the following theorem :

THEOREM2.2.1

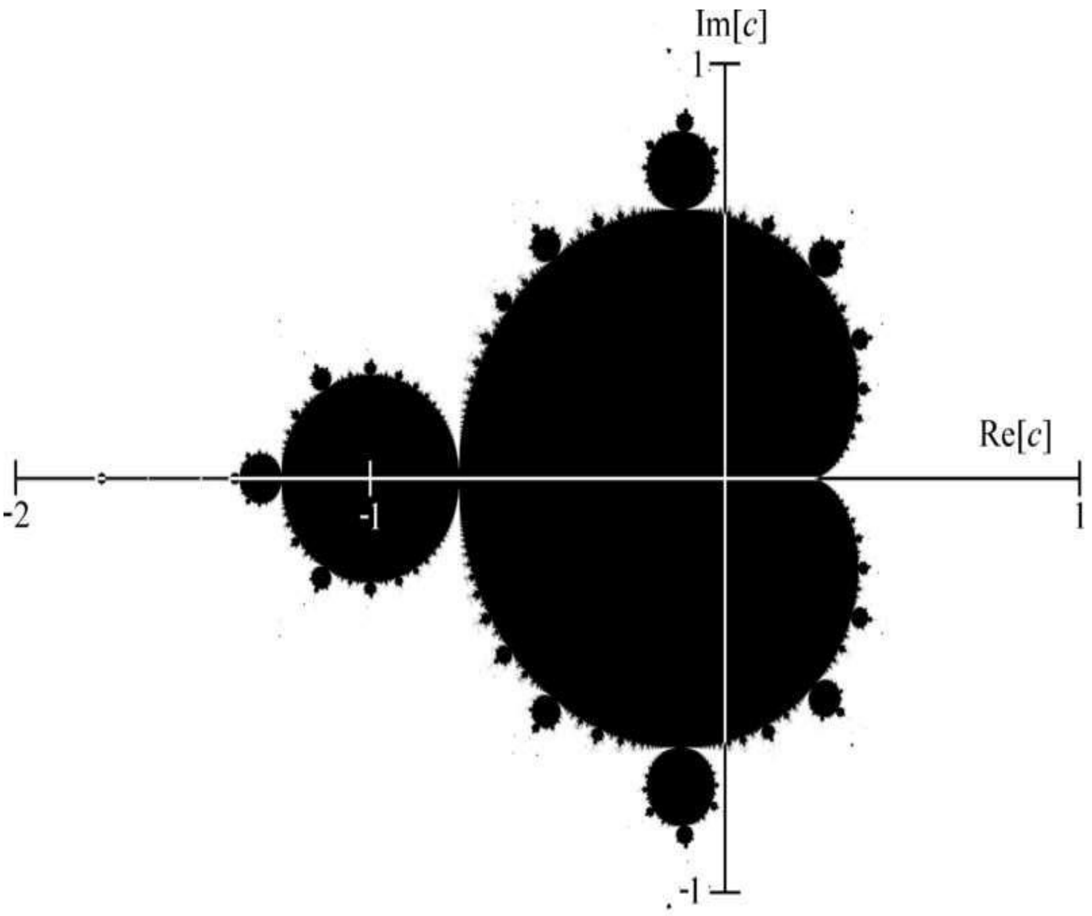
The Julia set corresponding to the function Q_c, for a specific c, is connected if and only if for all critical points z of Q_c , the orbit of z does not diverge to infinity.

A critical point of a function f is , as you recall from elementary calculus,a number z such that $f'(z)=0$. Since our functions are of the form $Q_c(z)=z^2+c$, the only critical point of each is z=0. Thus, the previous theorem leads to the following corollary making the determination of a Mandelbrot set far easier.

COROLLARY

A complex number c is in the Mandelbrot set if and only if the value 0 is in the filled Julia set of Q_c.

Graphical representations of the Mandelbrot set usually show the set itself in black. This mean a point c in the complex plane is colored black if the orbit $Q_c(0)=c$, $Q_c(Q_c(0))=c^2+c$, $Q_c(Q_c(Q_c(0)))=(c^2+c)^2+c$, etc., is bounded. If on the other hand, the values in this orbit diverge to infinity, or, in practice if they become greater than 2 in absolute value (in which case the orbit will diverge to infinity) , the iteration is broken off and the point c is colored white. In some cases, white is not used for points in this second set ; instead, gradations of color are used to indicate how long the iteration proceeded before being stopped. For example, if the iteration stopped after 10 steps(because the value became 2 or greater), the point is colored red, after 20 steps, orange, etc . These colors then show types of contours in the area outside the actual Mandelbrot set and create the magnificent colored pictures you may have seen. Using elementary properties of complex numbers, it can be shown that the Mandelbrot set lies entirely within the region in the c-plane where $|c|<2$ as shown in the figure below.



CHAPTER 3

FRACTAL DIMENSION

The fractal dimension of a set is a number that tells how densely the set occupies the metric space in which it lies. It is invariant under various stretching and squeezings of the underlying space.

Applying traditional methods of size measurement to highly irregular fractals leads to meaningless results. Instead, Mandelbrot and others discovered that to make any meaningful statement about the size of a fractal, they needed to resort to assigning it a dimension value ; but in order to do so , the concept of dimension had to be expanded.

SELF-SIMILARITY

Fractals are self similar at any level of magnification; many things around us look the same way no matter how you magnify them. When parts of some objects are similar to the entire object, we call it self-similar.

3.1- SELF-SIMILARITY DIMENSION

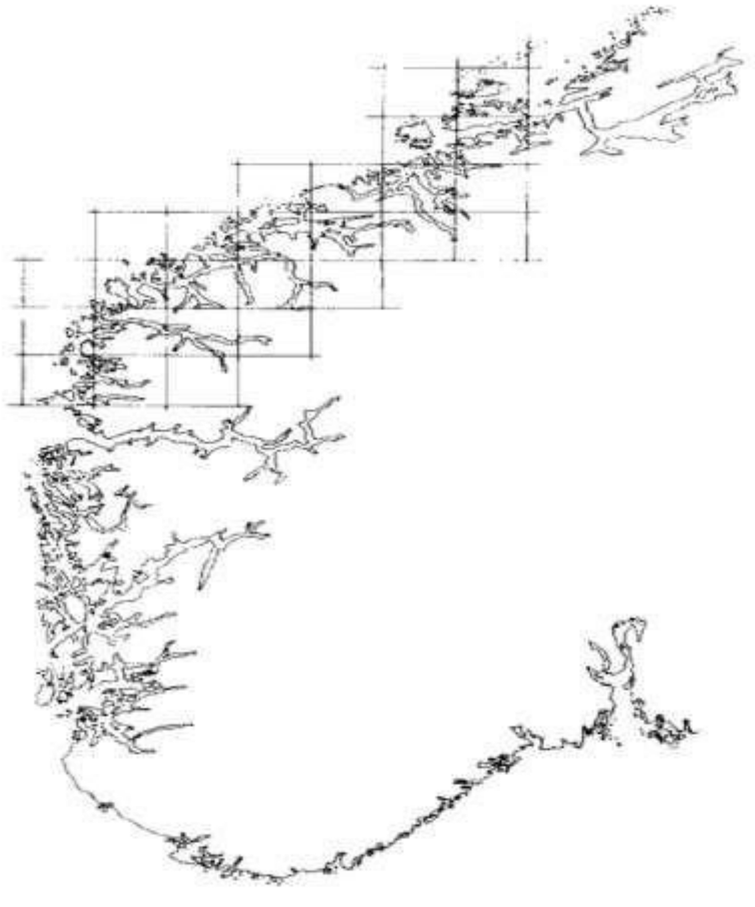
To assign fractals a self-similarity dimension, it is helpful to consider how segments, squares, and cubes can be tiled with a number of smaller tiles such that magnification of each tile by an integer scaling factor (using the same scaling factor for each tile) results in an object congruent to the original. To illustrate this, note that a segment can be tiled using two segment-shaped tiles (meeting at the midpoint of the original segment) so that magnification of each tile by the scaling factor 2 creates a segment congruent to the original. Similarly, a square can be tiled by four square-shaped tiles so that magnification of each tile by the scaling factor 2 (doubling each side) creates a square congruent to the original.

3.2 - BOX DIMENSION

Self-similarity dimension applies only to sets that are strictly self-similar, there are more generalized dimensions that can be applied to sets that are only "approximately" self-similar, including natural fractals like coastlines. One of these generalizations that move in the direction of the more esoteric Hausdorff-Besicovitch dimension is called box dimension. Here the term box refers to a segment, a square, or a cube, that is,a d-cube of the appropriate dimension d. To understand how box dimension generalizes self-similarity dimension, recall that the self-similarity dimension d of a set A is given by the equation $N=S^d$. Where s is the scaling-factor and N is the number of tiles in an s-scale tiling of A. Solving for d yields

Self-similarity dimension: $d_s = \frac{\ln N}{\ln s}$ [1]

When a set A is strictly self-similar and we have determined an appropriate scaling factor s, it is possible to tile the set with congruent s-tiles. Using the number of these tiles as N in Equation [1]above,we can immediately compute the fractal dimension of A. However,when A is not strictly self-similar, we cannot tile it with congruent "shrunken" copies of itself. So in good mathematical fashion, we approximate such a covering. To do so, we do not attempt to use smaller versions of the original set, but instead choose a box-shaped set with a side length ℓ and place a grid of these boxes over the set A. The dimension d of the box chosen depends on the nature of the set A. For example, even though it may seem that the appropriate box shape for any curve should be that of a segment, curves that are extremely "wiggly" are usually covered with square grids as shown below.



With the grid in place, we count the number of boxes that contain at least some portion of the set A. Then we reduce the side-length $\ell$and repeat this procedure with the same box shape. Clearly, the number of boxes required varies as ℓ changes since, as we reduce the side length of our boxes to achieve better fits, the number of "covering" boxes will generally increase.

We use the notation N(ℓ) to represent the number of covering boxes of side-length ℓ. In theory this process is iterated over and over as ℓ continues to shrink, thus explaining the need for the limit in the definition below.

DEFINITION 3.2.1

For a bounded set A, let N(ℓ) denote the minimum number of boxes of length $\ell > 0$ required to cover A. Then the box dimension of A is given by

Box dimension : d_B = limℓ tends to 0 (ln N(ℓ) / ln (1/ℓ)). [2]

Notice that the box dimension of an object is defined only when the limit in the above definition exists. And even in cases where the limit does exist, its value may not be obvious. However, the definition does give us a way to estimate the box dimension by evaluating the quotient in Equation [2] for several length ℓ . In practice, data points (ln(1/ℓ),ln N(ℓ)) are plotted and linear regression is used to find the line of best fit for the data. The slope of this line is then used as the box dimension.

CHAPTER 4

ITERATED FUNCTION SYSTEMS

A fractal set generally contains infinitely many points whose organization is so complicated that it is not possible to describe the set by specifying directly where each point in it lies. Instead, the set may be defined by 'the relations between the pieces'.

The examples of fractals, namely, the Koch curve and various sierpinski sets, are all strictly self-similar, that is, each can be tiled with congruent tiles where the tiles can be mapped onto the original using similarities with the same scaling factor ; or inversely, the original object can be mapped onto the individual tiles using similarities with a common scaling factor.

However as the above quotation indicates, there are so called fractals with such complexity that they are not self-similar in this strict sense. To construct objects with such complicated organizations would seem to require very involved procedures.However, the key to their construction is that the “relations between the pieces” of such fractals can be described using relatively small sets of the affine transformations .

4.1- AFFINE TRANSFORMATION

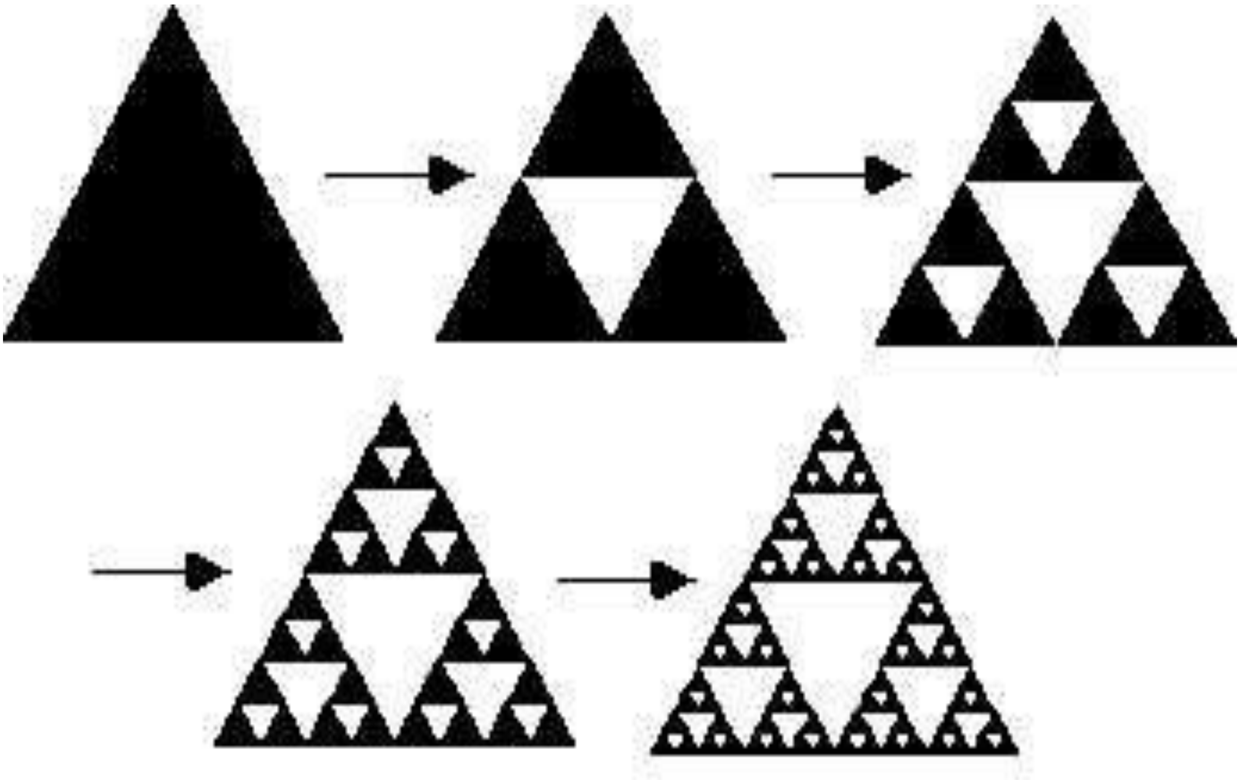
Affine transformations are linear transformations. Affine transformation is composition of rotation, translation, dilations and shears. An affine transformation do not preserve angles or length. Two or more successive transformations can be applied on the image with the use of affine transformation.

A transformation $w:R^2 \rightarrow R^2$ of the form $w(x,y) = (ax+by+e, cx+dy+f)$,Where a,b,c,d,e and f are real numbers, is called a (two dimensional) affine transformation. Using equivalent notations:

$$T\begin{pmatrix}x\\y\end{pmatrix} = s\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} + \begin{pmatrix}e\\f\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

A SIERPINSKI INTRODUCTION

To begin our consideration of fractal generating transformations, we will find a set of three so called sierpinski transformations that can be used in combination to generate the sierpinski triangle.



4.2 – IFS BASIC

IFS is generating fractal by repeatedly applying transformations (such as rotation and reflection) to points.

The set of sierpinski transformations is an example of an iterated function system.

DEFINITION 4.2.1

An iterated function system (IFS) is a finite set of contractive affine transformations.

DETERMINISTIC ITERATION

In the case of deterministic iteration, it is customary to use a set of multiple points as the initial input. We will represent such an initial set using the boldface notation $\mathbf{A_0}$ .The transformations of the IFS are then iteratively applied in a stage-by-stage procedure in which the output of any stage is considered to be a collage, or union, of the images produced by simultaneously applying each of the transformations to the output from the previous stage. When $\mathbf{A_0}$ is a multiple-point set, it follows that the images (sometimes called d-images to indicate their deterministic generation) will also be multiple-point sets which we will denote **An** . All of this is formalized in definition below.

DEFINITION 4.2.2

Let$T_1,...,T_K$ be the transformations of an IFS and let $\mathbf{A_0}$ bean initial set in the common domain of these transformations.Then$\mathbf{A_1}$, the first d-image of $\mathbf{A_0}$ under the IFS, consists of the following union of sets obtained by applying each of the transformations T_i once:
$\mathbf{A_1}=T_1(\mathbf{A_0})UT_2(\mathbf{A_0})U...T_K(\mathbf{A_0})$. And similarly $\mathbf{A_n}$, the nth d-image of A0 under the IFS, is given by : $\mathbf{A_n}=T_1(\mathbf{A_{n-1}})UT_2(\mathbf{A_{n-1}})U....UT_K(\mathbf{A_{n-1}})$. The process of finding these d-images is called deterministic iteration of the IFS and the sequence of dimages$\{\mathbf{A_0},\mathbf{A_1},\ldots\ldots\ldots..,\mathbf{A_n},\ldots..\}$ is the d-orbit of $\mathbf{A_0}$ under the IFS.

THEOREM 4.2.1

Let I be an IFS with domain **S**, a complete metric space, and assume$\mathbf{A_0}\subset$Let I be an IFS with domain **S ,**a complete metric space and assume $\mathbf{A_0}$C S . Then the d-orbit of $\mathbf{A_0}$ under I will have a unique limit set $\mathbf{A_\infty}$. Furthermore, the set $\mathbf{A_\infty}$ is independent of the initial set $\mathbf{A_0}$ and invariant under I. $\mathbf{A_\infty}$ is called the attractor of I.

Thus, just as each IFS transformation T has an invariant point that can be found by finding the limit point of the orbit of any point$\mathbf{A_0}$under T, the IFS itself has an invariant set that can be found by finding the limit set of the d-orbit of any initial set $\mathbf{A_0}$ under the IFS. And, as the name attractor suggests, this invariant set $\mathbf{A_\infty}$ is attracting, that is, if we deterministically apply the IFS to an initial set, we obtain a sequence of d-images that always "shrink" to this same unique $\mathbf{A_\infty}$.So, the IFS consisting of the two Cantor transformations does generate a unique Cantor set and the IFS consisting of the three Sierpinski transformations generates a unique Sierpinski triangle; where both can be approximated by d-image sets $\mathbf{A_n}$ with the approximation becoming more accurate as n increases.

RANDOM ITERATION

As noted above the sierpinski triangle can be generated from any initial multiple point set by deterministic iteration of the three sierpinski transformations . However, the chaos game also appears to generate the sierpinski triangle using a step-by-step procedure where each step involves randomly choosing one of these same transformations and applying it to a single point. At first glance , using deterministic iteration on an initial multiple point set appears to generate an image with a greater resemblance to the sierpinski triangle with less effort than the chose game . But a closer examination indicates the immensity of the memory requirement for the deterministic approach . Deterministic generation of the image . An requires that every transformation in the system be applied to all the point in the set $\mathbf{A_{n-1}}$; thus requiring storage for the location of each point in $\mathbf{A_{n-1}}$. By comparison the chose game requires storage for any one point ; since at each step one of the sierpinski transformation is applied to a single point to produce one image point ; the image point (sometimes called an r-image to indicate its random generation) is then added to the previous graphically display . In other words the graphical display at any step shows the accumulation of all previously generated image point.

DEFINITION 4.2.3

Let T_1 ………. T_k be the transformations of an IFS , and X_0 a given point in the common domain of these transformations . Then the nth r-image of X_0 is $X_n = T_{ni}(X_{n-1})$ with n chosen randomly (with pre-assigned probability) from the set (1,2,………..k) . The process of finding these r-image is called random iteration of the IFS and the infinite sequence of r-image (X_0,X_1,……X_n……)is an r-0rbit of X_0 under the IFS .

As you have noticed in playing the chaos game , when the initial point X_0 is chosen arbitrarily the first few r-image of X_0 may not lie in the alternate of the IFS in this case , the sierpinski triangle . However, when X_0 is an invariant point of one of the transformation in the IFS, we can show that all r-image of X_0 are element of $A\infty$ the attractors of the IFS. And as n increases , the finite r-orbit (X_0,X_1,…….X_n) appear to 'fill out' the attractor on an IFS is dependent only on the transformation involved and not on the initial set, this result may unseen less than surprising .But the difference in the sequence of transformation application between random and deterministic iteration means the result is not automatic. In fact , it is necessary to confirm that the elements in an r-orbit of X_0 under the IFS from a dense covering of $\mathbf{A_\infty}$**.**

THEOREM 4.2.2

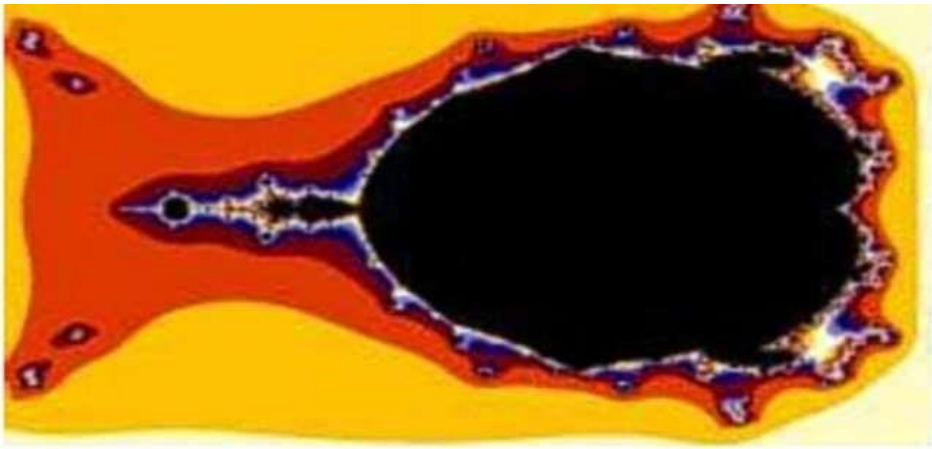
If I is an iterated function system with attractor $\mathbf{A_\infty}$**,** and $X_0$is an invariant point of one of the transformation in I , then the elements in an r-orbit of X_0 under I from a dense covering of $\mathbf{A_\infty}$.

Since the attractor of an IFS is a limiting set of an infinite process , its complete generation is only theoretically possible .However, with modern high speed computer , it is possible to generate approximation that are within the limits of resolution the available graphic display equipment . In practice, the computer depiction of an attractor $\mathbf{A_\infty}$ is usually generated by using the far more efficient method of computing a finite r-orbit $\mathbf{X_n}$ with n chosen sufficiently large so that the density of the covering of $\mathbf{A}_\infty$ by $\mathbf{X_n}$ is within the resolution of the computer.

4.3- ITERATING FUNCTIONS WITH COMPLEX NUMBERS AND THEMANDELBROT SET

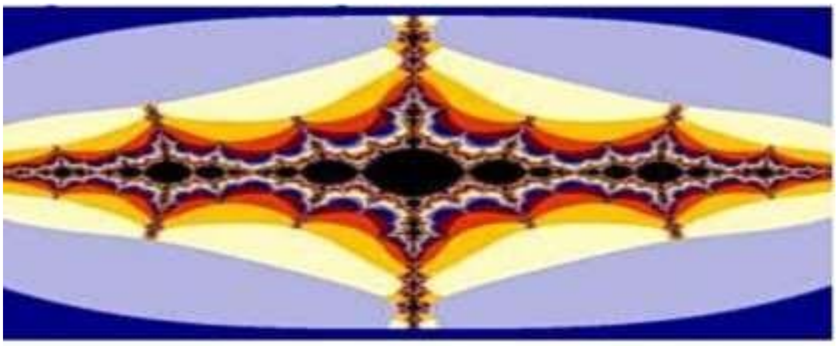
The basic principle of generating fractal employs the iterative formula $Q_c(z)=z^2+c$.Suppose the initial value of Z is 0.By putting the value of Z compute 0^2+c which gives c. After repeating the process we get $(c^2+c^2) + c$. And so forth. The list of complex number will be generated in this way. If these complex number (called the orbit of 0) get larger and larger (or further and further away from the origin) , then choice of c is NOT in the Mandelbrot set. But if this is not the case (the orbit stays “bonded”),then c is in the Mandelbort set .

The picture of Mandelbrot can be painted according to the following rule: Color c-value black if c does not escape to infinity, it means C lie with in the Mandelbrot set . Color a cvalue a different color if the orbit escape to infinity. Few iteration are required for the orbit of 0 under iteration of X^2+c to become far from the origin. Red points are followed in order by orange , yellow, green, blue , indigo and violet. That is we use the colors drawn from the light spectrum for points that are not in the Mandelbrot set. The important thing is: red points escape fastest, while violet points take the most iteration to go far away from the origin



orbit of Mandelbrot set

The filled Julia set for X^2+c is the collection of all seeds whose orbit does not escape to infinity under iteration of X^2+c.Thus there is a different filled Julia set for each c-value .



Corresponding Julia set of a seed inside the Mandelbrot se

CHAPTER 5

APPLICATIONS OF FRACTAL GEOMETRY

Fractal geometry is first and foremost a new "language" used to describe the complex forms found in nature. But while the elements of the "traditional language"-the familiar Euclidean geometry-are basic visible forms such as lines, circles and spheres, those of the new language do not lend themselves to direct observation. They are,namely, algorithms, which can be transformed into shapes and structures only with the help of computers. In addition, the supply of these algorithmic elements is inexhaustibly large; and they are capable of providing us with a powerful descriptive tool. Once this new language has been mastered, we can describe the form of a cloud as easily and precisely as an architect can describe a house using the language of traditional geometry.

Mandelbrot notes :"The importance of fractals lies in their ability to capture the essential features of very complicated and irregular objects and processes in a way that is susceptible to mathematical analysis" [Peterson, p.42] The fractal forms generated by computers are used by mathematicians and scientists to model a variety of natural phenomenon such as trees, coastlines, rivers,mountains, mineral veins, vascular systems, etc.

Nature has played a joke on the mathematicians. The pathological structures that the mathematicians invented to break loose from the 19th century naturalists turn out to be inherent in familiar objects all around us.

Furthermore, this wealth of applications threatens to overwhelm interest in the intriguing mathematics and the many open mathematical questions still remaining in fractal geometry. It seems now that deterministic fractal geometry is racing ahead into the serious engineering phase. Commercial applications have emerged in the areas of image compression, video compression, computer graphics and education.

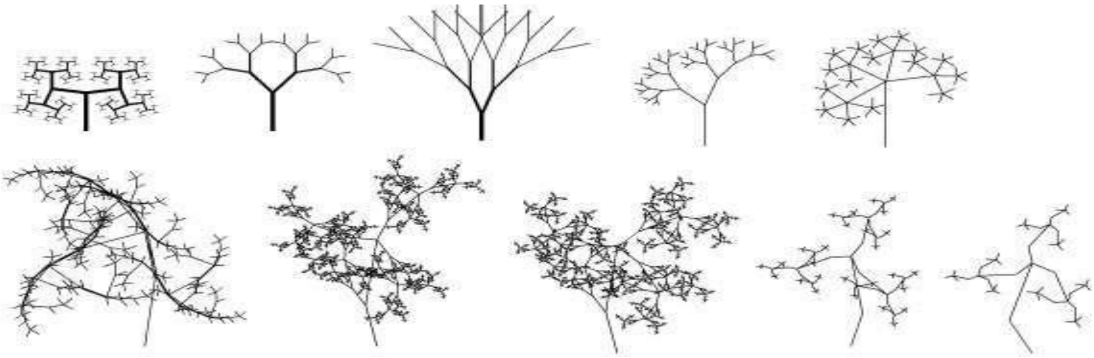
To many chaologists, the study of chaos and fractals is more than just a new field in science that unifies mathematics,theoretical physics, art, and computer Science it is a revolution. It is the discovery of a new geometry, one that describes the boundless universe we live in; one that is in constant motion, not as static images in textbooks. Today, many Scientists are trying to find applications for fractal geometry, from predicting stock market prices to making new discoveries in theoretical physics.Fractals have more and more applications science. The main reason is that they very often describe the real world better than traditional mathematics and physics.

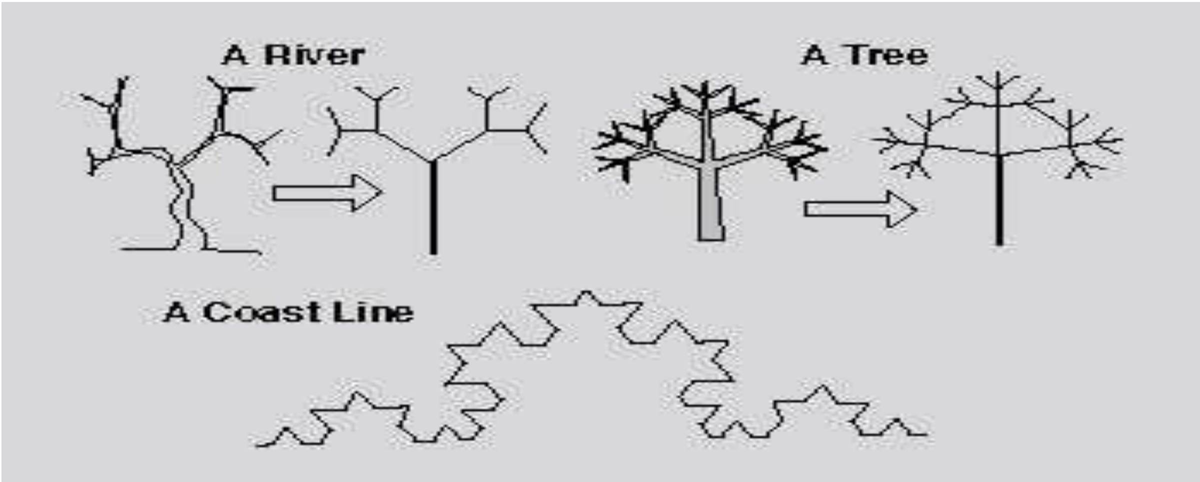
ASTRONOMY

Fractals will maybe revolutionize the way that the universe is seen.Cosmologists usually assume that matter is spread uniformly across space. But observation shows that this is not true.Astronomers agree with that assumption on "small" scales, but most of them think that the universe is smooth at very large scales. However, a dissident group of scientists claims that thestructure of the universe is fractal at all scales. If this new theory is proved to be correct, even the big bang models should be adapted.Some years ago we proposed a new approach for the analysis of galaxy and cluster correlations based on the concepts and methods of modern Statistical Physics. This led to the surprising result that galaxy correlations are fractal and not homogeneous up to the limits of the available catalogues. In the meantime many more redshifts have been measured and we have extended our methods also to the analysis of number counts and angular catalogues.The result is that galaxy structures are highly irregular and self-similar. The usual statistical methods,based on the assumption of homogeneity,are therefore inconsistent for all the length scales probed until now. A new,more general, conceptual framework is necessary to identify the real physical properties of these structures. But at present, Cosmologists need more data about the matter distribution in the universe to prove (or not) that we are living in a fractal universe.

NATURE

Take a tree, for example. Pick a particular branch and study it closely.Choose a bundle of leaves on that branch. To chaologists, all three of the objects described - the tree, the branch,and the leaves- are identical.To many the word chaos suggests randomness,unpredictability and perhaps even messiness. Chaos is actually very organized and follows certain patterns.The problem arises in finding these elusive and intricate patterns. One purpose of studying chaos through fractals is to predict patterns in dynamical systems that on the surface seem unpredictable. A system is a set of things,- an area of study -A Set of equations is a system, as well as more tangible things such as cloud formations,the changing weather, the movement of water currents, or animal migration patterns. Weather is a favourite example for many people. Forecasts are never totally accurate, and long-term forecasts,even for one week, can be totally wrong.This is due to minor disturbances in airflow, solar heating, etc. Each disturbance may be minor, but the change it create will increase geometrically with time. Soon, the weather will be far different than what was expected. With fractal geometry we can visually model much of what we witness in nature, the most recognized being coastlines and mountains. Fractals are used to model soil erosion and to analyze seismic patterns as welI. Seeing that so many facets of mother nature exhibit fractal properties, maybe the whole world around us is a fractal afterall!





COMPUTER SCIENCE

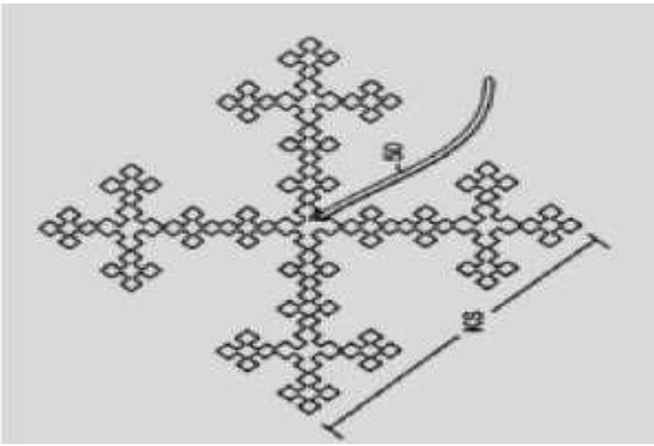
Actually, the most useful use of fractals in computer science is the fractal image compression. This kind of compression uses the fact that the real world is well described by fractal geometry. By this way, images are compressed much more than by usual ways (eg: JPEG or GIF file formats). An other advantage of fractal compression is that when the picture is enlarged, there is no pixelisation. The picture seems very often better when its size is increased.

FLUID MECHANICS

The study of turbulence in flows is very adapted to fractals. Turbulent flows are chaotic and very difficult to model correctly. A fractal representation of them helps engineers and physicists to better understand complex flows. Flames can also be simulated. Porous media have a very complex geometry and are well represented by fractal.This is actually used in petroleum science.

TELECOMMUNICATIONS

Anew application is fractal-shaped antennae that reduce greatly the size and the weight of the antennas. Fractenna is the company which sells these antennae.The benefits depend on the fractal applied, frequency of interest, and so on.In general the fractal parts produces 'fractal loading' and makes the antenna smaller for a given frequency of use.Practical shrinkage of 24 times are realizable for acceptable performance.Surprisingly high performance is attained.



SURFACE PHYSICS

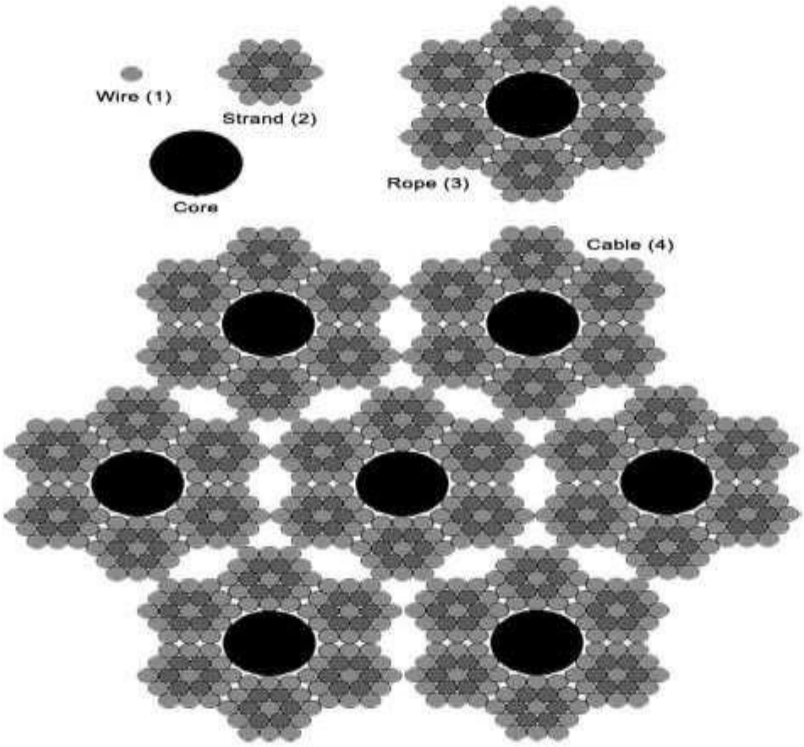
Fractals are used to describe the roughness of surfaces. A rough surface is characterized by a combination of two different fractals.

ARCHITECTURE

Most of the building created by man, has Euclidean shapes and pattern but building can be constructed by using fractal geometry. The reason of using fractal geometry in architecture is to mimic the patterns in Nature.

CABLE SAND BRIDGES

Fractal ideas can be used to make super-strong cables. This repetitive, fractal pattern in cables provides great strength. A steel cable is formed from a bundle of smaller cables which themselves are formed of smaller bundles, etc.



BACTERIA CULTURES

The spreading of bacteria can be modelled by fractals such as the diffusion fractals.

MEDICINE

With the use of modern medicine malfunctioning in the human body can be detected. Because human body is full of fractals,fractal math can be used to quantify, describe, diagnose and perhaps soon to help cure diseases. The fractal dimension of the lung appears to vary between healthy and sick lungs, potentially aiding in the automated detection of the disease. To diagnose Cancer, fractal analysis is helpful.

FRACTAL IN LANDSCAPES

Fractal landscapes is a very classic application of fractals. If structure of mountain is magnified we get more and more detail at each level of magnification. The structure of small part of mountain is same as structure of whole image. This is called self-similarity. Usually plasma fractals are used for the landscapes because they give the most realistic pictures.

FRACTAL MOLECULES

Fractal science can be used to study sequence of nucleotides which is called the DNA walk. The DNA walk is a graphical representation of the DNA sequence. These pattern are remarkably similar to Brownian motion.

FRACTAL IN POPULATION GROWTH

Fractals can be used to analyze the rapid growth of population of developing countries. Last century, Thomas Malthus came with a theory in which he said that with every generation, the population increases a certain amount of times depending on the growth rate. Mathematically, if we make r the percent growth rate, and P the population, our formula will become

new P= (1+r).P ....(1)

For example, if r= 1/2 the population will increase 50%, or become 1.5 times larger. According to this theory,the population will increase infinitely [18]. However, the population is really limited by natural resources, such as space and food. Let's pretend the maximum possible population the environment can hold is 1, so P is a number from 0 to 1. As the population gets closer to 1, the growth rate is going to decrease and get close to 0. We can achieve this by multiplying the growth rate by (1-P). This way, as P is getting closer to 1, the growth rate will be multiplied by a number that is getting close to 0. We now determined that the growth rate should really be r(1-P). If we use it in the above formula, we get

If we now do some algebra

new P=[1+r- rP].p........(2)

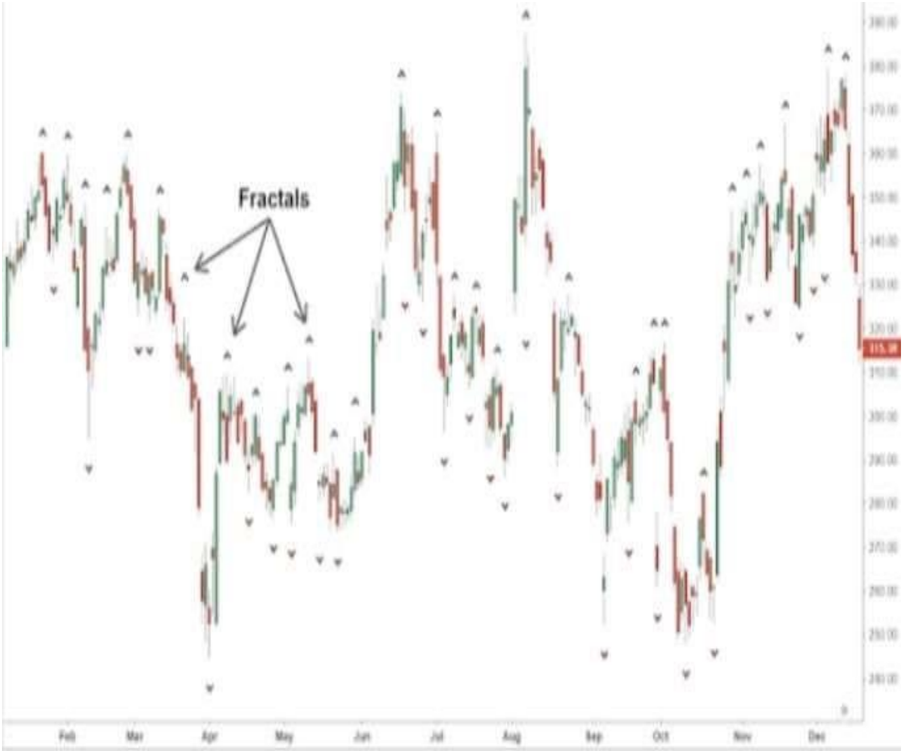
new P =p +rP- rp^2........(3)

new P = (1+ r). P-rp^2 ....(4)

Will now use this formula. Knowing this formula, it is easy to determine what the population becomes after a long period of time. For example, when r is between 0 and 2, the population becomes 1 and stays there, no matter what it was at the beginning. When it is 2.25, it will always end up jumping between 1.17 and 0.72. When r is 2.5, it ends up jumping between 1.22, 0.54, 1.16, and 0.70. When it is 2.5, it ends up jumping between 8 values, and when r gets higher,it jumps between 16 values. As we increase r, the number of these values doubles.

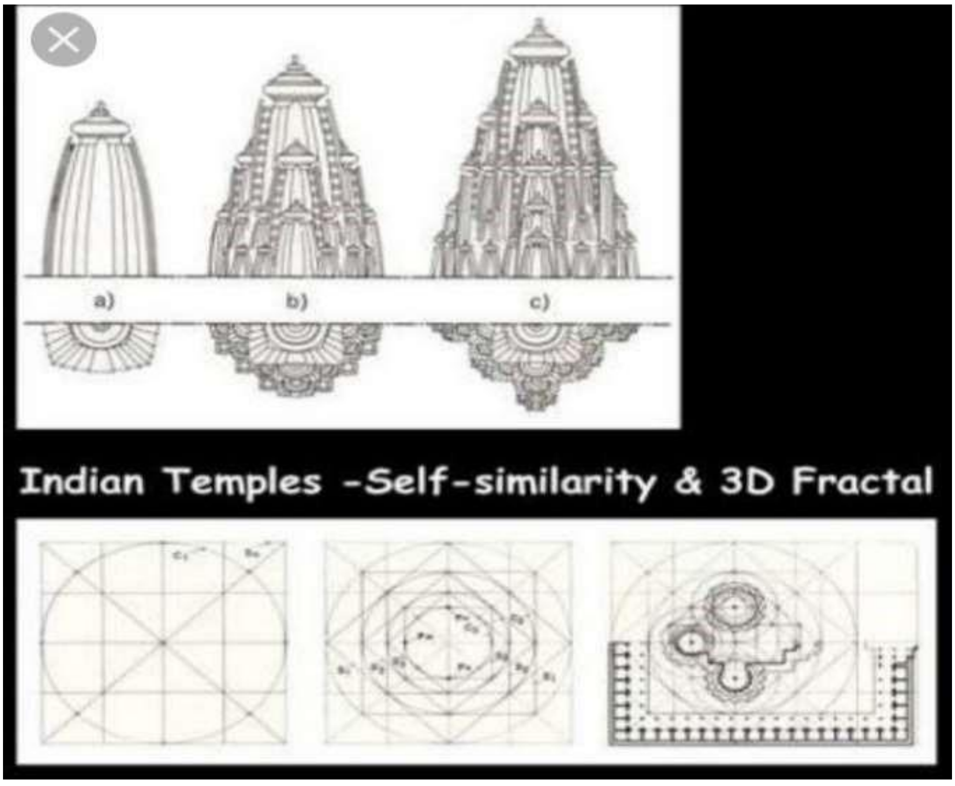
FRACTAL IN MARKET ANALYSIS

Benoit Mandelbrot introduced a new fractal theory which is helpful to analyze the market. After plotting price data of market for a month some rises and fall will be appeared in the graph. If this graph is plotted for weak or even for a day same rises and fall will be appear. This is self similar property of fractal. It is also called Brownian self-similarity.



FRACTAL IN ART

The concept of Fractal can be used to create pictures which are more complicated in nature and they have the property of self-similarity. Mandelbrot set suggested by benoit Mandelbrot is a good example of fractal science.



CHAPTER 6
CONCLUSION

Many object in the nature can be created by applying the concept of classical geometry like-lines, circles, conic sections,polygons, spheres, quadratic surface and so on. There are various objects of nature which can not be modelled by applying Euclidean geometry, hence there is need to deal with Such complicated and irregular object which can only be constructed by fractal geometry.

To generate such complicated object iteration process is required which is called iterated function system.

The term "fractal" was invented by Mandelbrot to describe geometric shapes that in simplistic terms can be described as very fractured.Fractals have always been associated with the term chaos . One author elegantly describes fractals as "the pattern of chaos" . Fractals depict chaotic behaviours , yet if one looks closely enough,it is always possible to spot glimpses of self-similarity within a fractal.The main property in every fractal object is self similarity. Upon magnification of a Fractal, we can find subsets of it that look like the whole figure. If we zoom on a picture of a mountain again and again we still see a mountain. This is the self similarity of fractal.

Ecologists have found fractal geometry to be an extremely useful tool for describing ecological systems. Many population, community, ecosystem, and landscape ecologists use fractal geometry as a tool to help define and explain the systems in the world around us. As with any scientific field, there has been some dissension in ecology about the appropriate level of study. For example, some organism ecologists think that anything larger than a single organism obscures the realty with too much detail. On the other hand, some ecosystem ecologists believe that looking at anything less than an entire ecosystem will not give meaningful results. In reality, both perspectives are correct. Ecologists must take all levels of organization into account to get the most out of a study. Fractal geometry is a tool that bridges the "gap" between different fields of ecology and provides a common language.

One of the most valuable aspects of fractal geometry, however, is the way that it bridges the gap between ecologists of differing fields. By providing a common language, fractal geometry allows ecologists to communicate and share ideas and concepts. As the information and computer age progress, with better and faster computers, fractal geometry will become an even more important tool for ecologists and biologists. Some future applications of fractal geometry to ecology include climate modelling, weather prediction,land management, and the creation of artificial habitats.

This paper has describes various fundamental concepts and properties of fractal geometry theory. Various methods for calculating fractal dimension has also been dicussed. This paper provides discussion for basic transformations, iterated function system and other theorems which are used in fractal generation process.

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