E 9293 (Pages : 3) Reg. No.................................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2014

Third Semester

Complementary Course—Statistics—PROBABILITY DISTRIBUTION

[Common to B.Sc. Mathematics, Physics and Computer Applications (Three main)]

[2013 Admissions]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions.*

1 mark each.

1. If X follows a binomial distribution (6, 0.3), what is the distribution of 6 – X ?
2. Define a Poisson distribution with parameter λ.
3. If $X \sim N(\mu, \sigma^2)$, what is the maximum probability occurring at $x = \mu$?
4. Find out the 7^{th} central moment of $N(\mu, \sigma^2)$.
5. What is the sum of two independent exponential r.v's with parameter θ ?
6. In case of one parameter gamma distribution. What is the relation connecting mean and variance ?
7. What is the distribution of the ratio of 2 independent standard normal r.vs ?
8. Write down the mean and variance of a Chi-square r.v. with 2 d.f.
9. State Tchebycheff's inequality.
10. Define discrete uniform distribution.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*

2 marks each.

11. Find the m.g.f. of binomial distribution (n, p).
12. Establish the additive property of Poisson distribution.
13. Define Beta type I and type II distributions.
14. Establish lack of memory property of geometric distribution.
15. State weak law of large numbers. Give an example where the weak law is obeyed.

Turn over

16. A r.v. X has mean = 3 and S.D. = 2 Find an upper bound to $p(|X-3|>4)$.

17. Find the mode of a binomial distribution for which mean = 4 and S.D. = $\sqrt{3}$.

18. Define parameter, statistic, sampling distribution and standard error.

19. Find the relationship between Chi-squares and F random variables.

20. Define a '*t*' statistic. Give an example.

21. What is the distributions of $y = \frac{ns^2}{\sigma^2}$, where $X \sim N(\mu, \sigma^2)$ and S^2 is the sample variance ? What is the mean and m.g.f. Y ?

22. State Lindeberg-Levy form of CLT. Mention its uses.

(8 × 2 = 16)

Part C (Description /Short Essay)

Answer any **six** *questions.*
4 marks each.

23. State and prove Bernoullis' law of large numbers.

24. Derive the m.g.f. of gamma distribution and hence obtain its mean and variance.

25. For a normal distribution 38% of the observation are below 64 and 12% over 92. Find the mean and variance.

26. Use CLT to find the least value of *n* if we require $p(\overline{X} > \mu + 0.281\,\sigma) = 0.05$.

27. If X~ an exponential distribution with parameter 1, obtain $p(|X-1|>2)$ by Tchebychev's inequality and compare it with the actual probability.

28. Obtain the sampling distribution of mean and variance of a sample of size '*n*' taken form $N(\mu, \sigma^2)$.

29. A r.v. X has a Chi-square distribution with *n* d.f. show that $E(\sqrt{X}) = \sqrt{2}\,\frac{\Gamma\frac{(n+1)}{2}}{\Gamma\left(\frac{n}{2}\right)}$.

30. If X_i is a r.v. which assumes values i and $-i$ with equal probabilities, show that the law of large numbers can not be applied to the sequence $X_1, X_2, \ldots$

31. State and prove Bernoullis' law of large numbers.

(6 × 4 = 24)

Part D (Essay)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. Define a normal probability law. What are its important characteristics ? Derive the mean and variance of $X \sim N(0, 1)$?

33. Show that a Poisson distribution tends to a normal distribution. Establish the recurrence relation satisfied by the central moments of a Poisson distribution.

34. (a) If $E(X) = 70$, $\sigma = 7$, how large a sample should be taken in order that $p\left\{\left|\overline{X} - 70\right| \leq 1\right\} \geq 0.99$.

    (b) Bring out the importance of CLT and weak law of large numbers.

35. (a) If $X_1, \ldots X_n$ are n independent r.vs each having gamma distribution with parameters (λ, n), obtain the distribution of $\sum_{i=1}^{n} X_i$.

    (b) Derive the distribution of the difference of two sample means each following normal distribution with identical variance (unknown) with small sample sizes.

(2 × 15 = 30)